

EUREKA MATH

Module 2

Name: _____

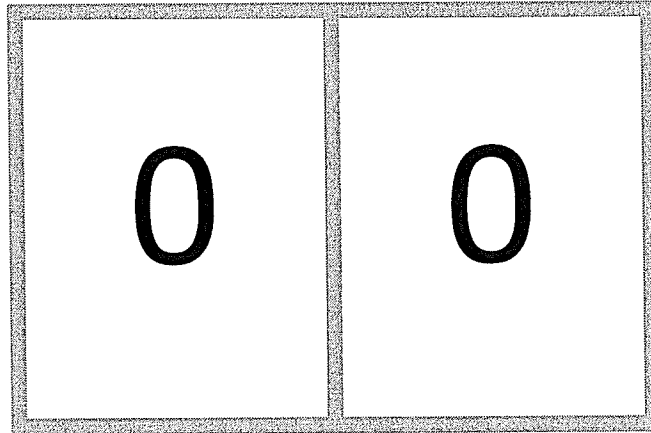
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19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475	494	513	532	551	570
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21	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525	546	567	588	609	630
22	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550	572	594	616	638	660
23	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575	598	621	644	667	690
24	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600	624	648	672	696	720
25	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700	725	750
26	26	52	78	104	130	156	182	208	234	260	286	312	338	364	390	416	442	468	494	520	546	572	598	624	650	676	702	728	754	780
27	27	54	81	108	135	162	189	216	243	270	297	324	351	378	405	432	459	486	513	540	567	594	621	648	675	702	729	756	783	810
28	28	56	84	112	140	168	196	224	252	280	308	336	364	392	420	448	476	504	532	560	588	616	644	672	700	728	756	784	812	840
29	29	58	87	116	145	174	203	232	261	290	319	348	377	406	435	464	493	522	551	580	609	638	667	696	725	754	783	812	841	870
30	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510	540	570	600	630	660	690	720	750	780	810	840	870	900

The Integer Cards

1	2	3	4
5	6	7	8
9	10	11	12

-1	-2	-3	-4
-5	-6	-7	-8
-9	-10	-11	-12

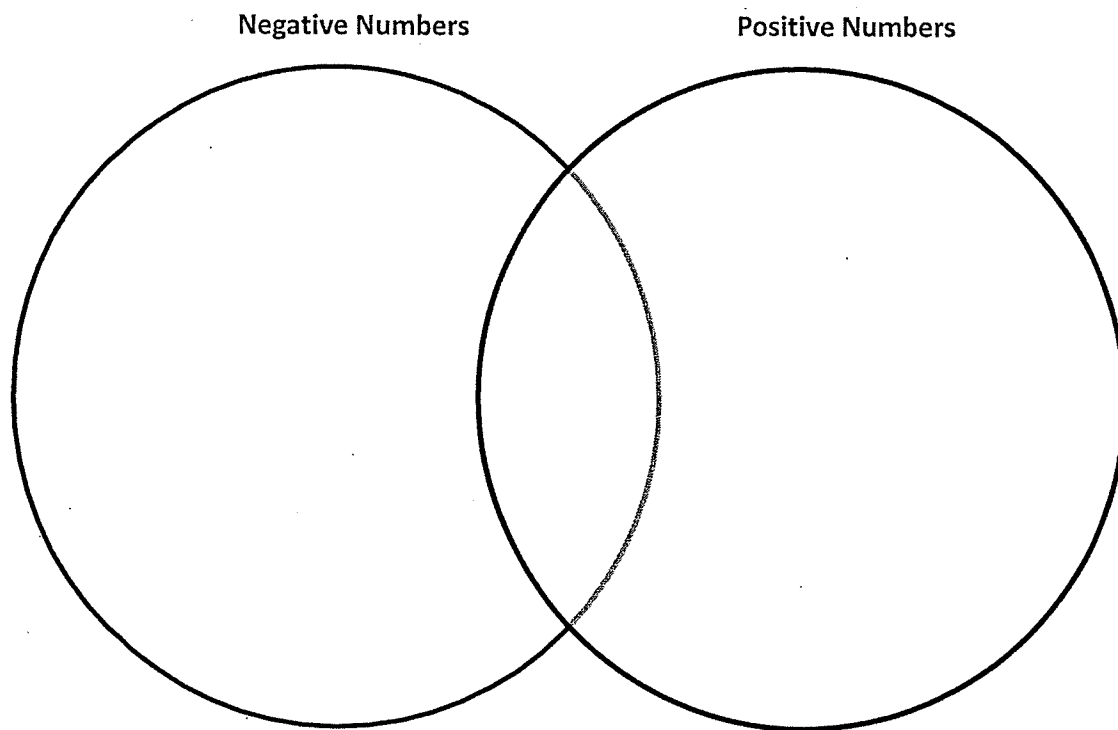


Lesson 1: Opposite Quantities Combine to Make Zero

Classwork

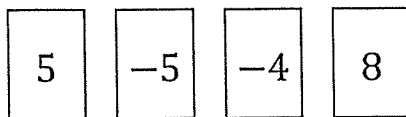
Exercise 1: Positive and Negative Numbers Review

With your partner, use the graphic organizer below to record what you know about positive and negative numbers. Add or remove statements during the whole-class discussion.



Example 3: Using the Integer Game and the Number Line.

What is the sum of the card values shown? Use the counting on method on the provided number line to justify your answer.

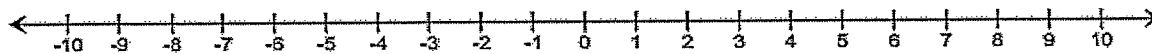


a. What is the final position on the number line? _____

b. What card or combination of cards would you need to get back to 0? _____

Exercise 2: The Additive Inverse

Use the number line to answer each of the following questions.



a. How far is 7 from 0 and in which direction? _____

b. What is the opposite of 7? _____

c. How far is -7 from 0 and in which direction? _____

- d. Thinking back to our previous work, explain how you would use the counting on method to represent the following: While playing the Integer Game, the first card selected is 7, and the second card selected is -7 .
- e. What does this tell us about the sum of 7 and its opposite, -7 ?
- f. Look at the curved arrows you drew for 7 and -7 . What relationship exists between these two arrows that would support your claim about the sum of 7 and -7 ?
- g. Do you think this will hold true for the sum of any number and its opposite? Why?

Property: For every number a , there is a number $-a$ so that $a + (-a) = 0$ and $(-a) + a = 0$.

The *additive inverse of a number* is a number such that the sum of the two numbers is 0. The opposite of a number satisfies this definition: For example, the opposite of 3 is -3 , and $3 + (-3) = 0$. Hence -3 is the additive inverse of 3.

The property above is usually called the existence of additive inverses.

Exercise 3: Playing the Integer Game

Play the Integer Game with your group. Use a number line to practice counting on.

Name _____

Date _____

Lesson 1: Opposite Quantities Combine to Make Zero

Exit Ticket

1. Your hand starts with the 7 card. Find three different pairs that would complete your hand and result in a value of zero.

7		
7		
7		

2. Write an equation to model the sum of the situation below.

A hydrogen atom has a zero charge because it has one negatively charged electron and one positively charged proton.

3. Write an equation for each diagram below. How are these equations alike? How are they different? What is it about the diagrams that lead to these similarities and differences?

Diagram A:

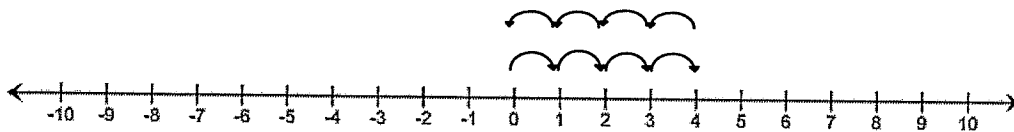
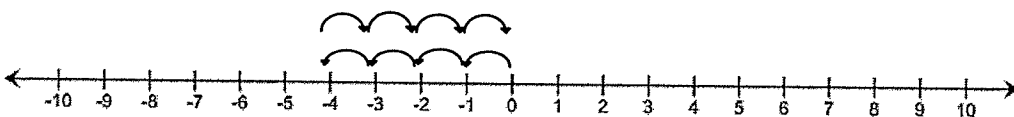


Diagram B:



Name _____

Date _____

Lesson 1: Opposite Quantities Combine to Make Zero

Lesson Summary

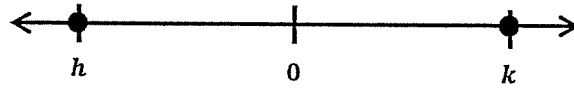
- Add a positive number to a number by counting up from that number, and add a negative number to a number by counting down from that number.
- An integer plus its opposite sum to zero.
- The opposite of a number is called the additive inverse because the two numbers' sum is zero.

Problem Set

For Problems 1 and 2, refer to the Integer Game.

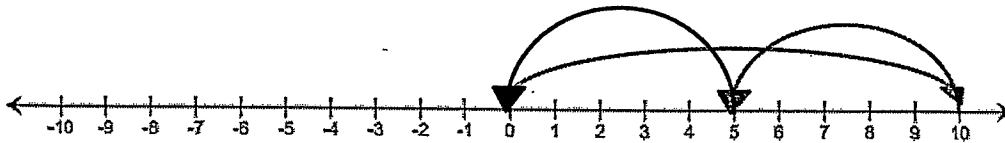
1. You have two cards with a sum of (-12) in your hand.
 - a. What two cards could you have?
 - b. You add two more cards to your hand, but the total sum of the cards remains the same, (-12) . Give some different examples of two cards you could choose.
2. Choose one card value and its additive inverse. Choose from the list below to write a real-world story problem that would model their sum.
 - a. Elevation: above and below sea level
 - b. Money: credits and debits, deposits and withdrawals
 - c. Temperature: above and below 0 degrees
 - d. Football: loss and gain of yards

3. On the number line below, the numbers h and k are the same distance from 0. Write an equation to express the value of $h + k$. Explain.



4. During a football game, Kevin gained five yards on the first play. Then he lost seven yards on the second play. How many yards does Kevin need on the next play to get the team back to where they were when they started? Show your work.

5. Write an addition number sentence that corresponds to the arrows below.



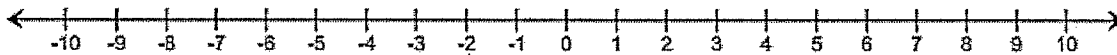
Lesson 2: Using the Number Line to Model the Addition of Integers

Classwork

Exercise 1: Real-World Introduction to Integer Addition

Answer the questions below.

- Suppose you received \$10 from your grandmother for your birthday. You spent \$4 on snacks. Using addition, how would you write an equation to represent this situation?
- How would you model your equation on a number line to show your answer?

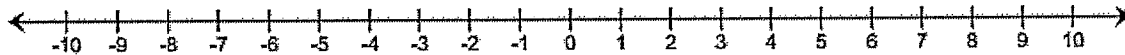


Example 1: Modeling Addition on the Number Line

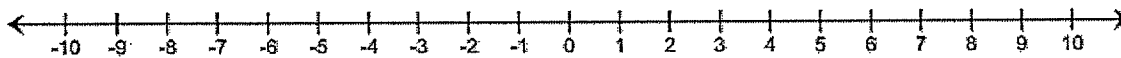
Complete the steps to find the sum of $-2 + 3$ by filling in the blanks. Model the equation using straight arrows called *vectors* on the number line below.

- Place the tail of the arrow on _____.
- Draw the arrow 2 units to the left of 0, and stop at _____. The direction of the arrow is to the _____ since you are counting down from 0.
- Start the next arrow at the end of the first arrow, or at _____.
- Draw the second arrow _____ units to the right since you are counting up from -2 .
- Stop at _____.

- f. Circle the number at which the second arrow ends to indicate the ending value.



- g. Repeat the process from parts (a)–(f) for the expression $3 + (-2)$.



- h. What can you say about the sum of $-2 + 3$ and $3 + (-2)$? Does order matter when adding numbers? Why or why not?

Example 2: Expressing Absolute Value as the Length of an Arrow on the Real Number Line

- a. How does absolute value determine the arrow length for -2 ? Use the number line provided to support your answer.



- b. How does the absolute value determine the arrow length for 3? Use the number line provided to support your answer.

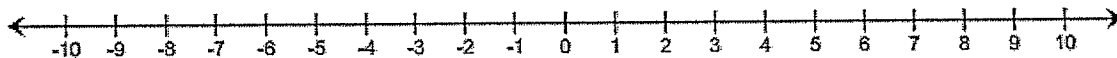


- c. Describe how the absolute value helps you represent -10 on a number line.

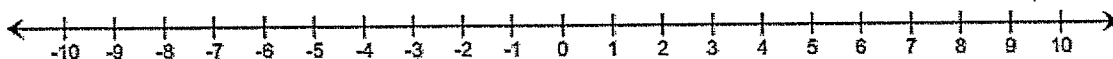
Exercise 2

Create a number line model to represent each of the expressions below.

a. $-6 + 4$



b. $3 + (-8)$



Example 3: Finding Sums on a Real Number Line Model

Find the sum of the integers represented in the diagram below.



- Write an equation to express the sum.
- What three cards are represented in this model? How did you know?
- In what ways does this model differ from the ones we used in Lesson 1?
- Can you make a connection between the sum of 6 and where the third arrow ends on the number line?
- Would the sum change if we changed the order in which we add the numbers, for example, $(-2) + 3 + 5$?
- Would the diagram change? If so, how?

Exercise 3

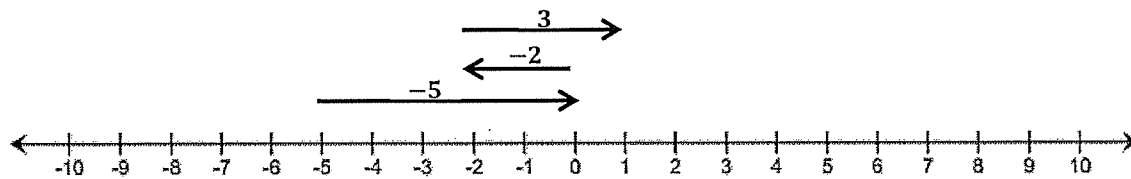
Play the Integer Game with your group. Use a number line to practice counting on.

Name _____

Date _____

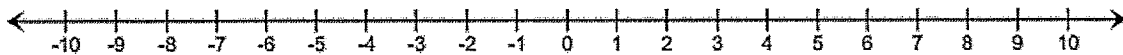
Lesson 2: Using the Number Line to Model the Addition of Integers

Exit Ticket



Jessica made the addition model below of the expression $(-5) + (-2) + 3$.

- Do the arrows correctly represent the numbers that Jessica is using in her expression?
- Jessica used the number line diagram above to conclude that the sum of the three numbers is 1. Is she correct?
- If she is incorrect, find the sum, and draw the correct model.



- Write a real-world situation that would represent the sum.

Name _____

Date _____

Lesson 2: Using the Number Line to Model the Addition of Integers

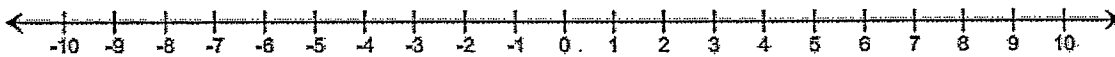
Lesson Summary

- On a number line, arrows are used to represent integers; they show length and direction.
- The length of an arrow on the number line is the absolute value of the integer.
- Adding several arrows is the same as combining integers in the Integer Game.
- The sum of several arrows is the final position of the last arrow.

Problem Set

Represent Problems 1–3 using both a number line diagram and an equation.

1. David and Victoria are playing the Integer Card Game. David drew three cards, -6 , 12 , and -4 . What is the sum of the cards in his hand? Model your answer on the number line below.

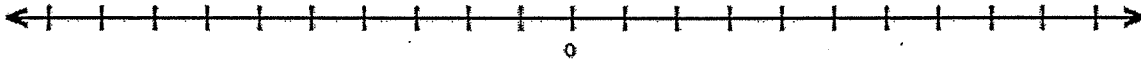


2. In the Integer Card Game, you drew the cards, 2, 8, and -11 . Your partner gave you a 7 from his hand.
- a. What is your total? Model your answer on the number line below.



- b. What card(s) would you need to get your score back to zero? Explain. Use and explain the term *additive inverse* in your answer.

3. If a football player gains 40 yards on a play, but on the next play, he loses 10 yards, what would his total yards be for the game if he ran for another 60 yards? What did you count by to label the units on your number line?



4. Find the sums.

a. $-2 + 9$

b. $-8 + -8$

c. $-4 + (-6) + 10$

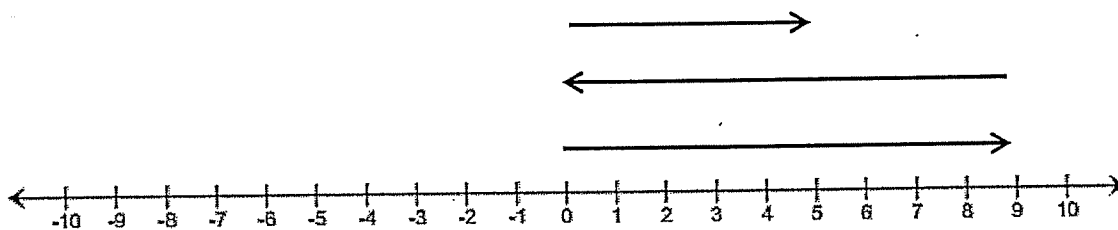
d. $5 + 7 + (-11)$

5. Mark an integer between 1 and 5 on a number line, and label it point Z . Then, locate and label each of the following points by finding the sums.

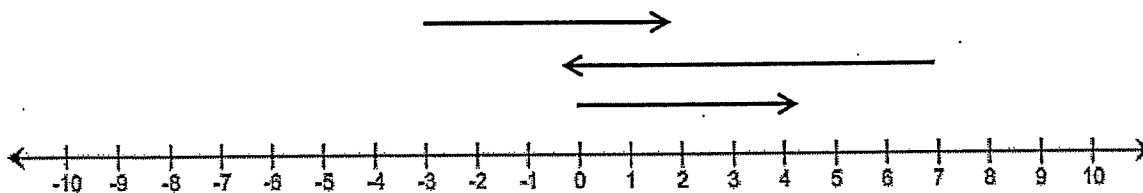


- Point A : $Z + 5$
- Point B : $Z + (-3)$
- Point C : $(-4) + (-2) + Z$
- Point D : $-3 + Z + 1$

6. Write a story problem that would model the sum of the arrows in the number diagram below.



7. Do the arrows correctly represent the equation $4 + (-7) + 5 = 2$? If not, draw a correct model below.

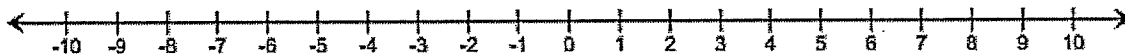


- d. Is the sum to the right or left of 54 on the horizontal number line? Above or below on a vertical number line?
- e. Given the expression $14 + (-3)$, determine, without finding the sum, the distance between 14 and the sum. Explain.
- f. Is the sum to the right or left of 14 on the number line? Above or below on a vertical number line?

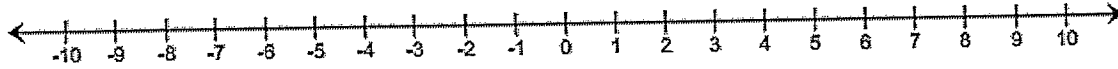
Exercise 2

Work with a partner to create a horizontal number line model to represent each of the following expressions. What is the sum?

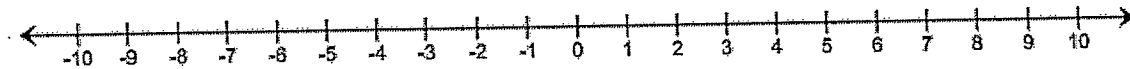
a. $-5 + 3$



b. $-6 + (-2)$



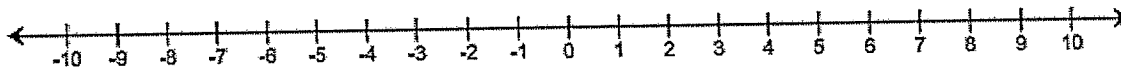
c. $7 + (-8)$

**Exercise 3: Writing an Equation Using Verbal Descriptions**

Write an equation, and using the number line, create an *arrow diagram* given the following information:

The sum of 6 and a number is 15 units to the left of 6 on the number line.

Equation:



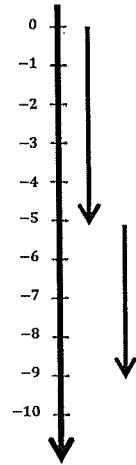
Name _____

Date _____

Lesson 3: Understanding Addition of Integers

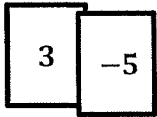
Exit Ticket

- Refer to the diagram to the right.
 - Write an equation for the diagram to the right. _____
 - Find the sum. _____
 - Describe the sum in terms of the distance from the first addend. Explain.
 - What integers do the arrows represent? _____

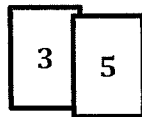


- Jenna and Jay are playing the Integer Game. Below are the two cards they selected.
 - How do the models for these two addition problems differ on a number line? How are they the same?

Jenna's Hand

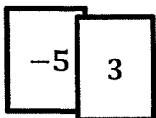


Jay's Hand

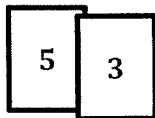


- If the order of the cards changed, how do the models for these two addition problems differ on a number line? How are they the same?

Jenna's Hand



Jay's Hand



Name _____

Date _____

Lesson 3: Understanding Addition of Integers

Lesson Summary

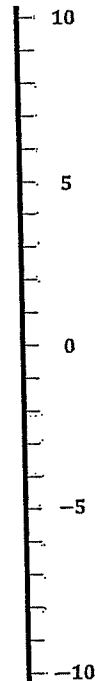
- Adding an integer to a number can be represented on a number line as counting up when the integer is positive (just like whole numbers) and counting down when the integer is negative.
- Arrows can be used to represent the sum of two integers on a number line.

Problem Set

- Below is a table showing the change in temperature from morning to afternoon for one week.
 - Use the vertical number line to help you complete the table. As an example, the first row is completed for you.

Change in Temperatures from Morning to Afternoon

Morning Temperature	Change	Afternoon Temperature	Equation
1°C	Rise of 3°C	4°C	$1 + 3 = 4$
2°C	Rise of 8°C		
-2°C	Fall of 6°C		
-4°C	Rise of 7°C		
6°C	Fall of 9°C		
-5°C	Fall of 5°C		
7°C	Fall of 7°C		



- Do you agree or disagree with the following statement: "A rise of -7°C " means "a fall of 7°C "? Explain. (Note: No one would ever say, "A rise of -7 degrees"; however, mathematically speaking, it is an equivalent phrase.)

For Problems 2–3, refer to the Integer Game.

2. Terry selected two cards. The sum of her cards is -10 .

a. Can both cards be positive? Explain why or why not.

b. Can one of the cards be positive and the other be negative? Explain why or why not.

c. Can both cards be negative? Explain why or why not.

3. When playing the Integer Game, the first two cards you selected were -8 and -10 .

a. What is the value of your hand? Write an equation to justify your answer.

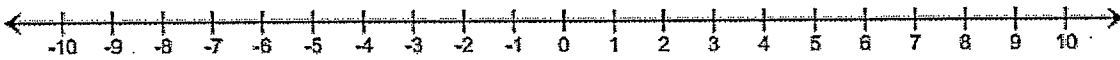
b. For part (a), what is the distance of the sum from -8 ? Does the sum lie to the right or left of -8 on the number line?

c. If you discarded the -10 and then selected a 10 , what would be the value of your hand? Write an equation to justify your answer.

4. Given the expression $67 + (-35)$, can you determine, without finding the sum, the distance between 67 and the sum? Is the sum to the right or left of 67 on the number line?

5. Use the information given below to write an equation. Then create an *arrow diagram* of this equation on the number line provided below.

The sum of -4 and a number is 12 units to the right of -4 on a number line.

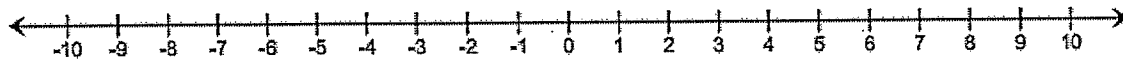


Lesson 4: Efficiently Adding Integers and Other Rational Numbers

Classwork

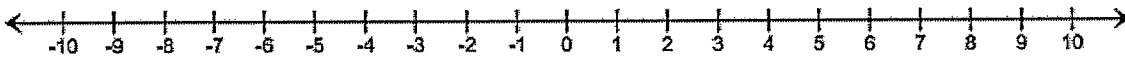
Example 1: Rule for Adding Integers with Same Signs

- a. Represent the sum of $3 + 5$ using arrows on the number line.



- i. How long is the arrow that represents 3?
- ii. What direction does it point?
- iii. How long is the arrow that represents 5?
- iv. What direction does it point?
- v. What is the sum?
- vi. If you were to represent the sum using an arrow, how long would the arrow be, and what direction would it point?

- vii. What is the relationship between the arrow representing the number on the number line and the absolute value of the number?
- viii. Do you think that adding two positive numbers will always give you a greater positive number? Why?
- b. Represent the sum of $-3 + (-5)$ using arrows that represent -3 and -5 on the number line.



- i. How long is the arrow that represents -3 ?
- ii. What direction does it point?
- iii. How long is the arrow that represents -5 ?
- iv. What direction does it point?
- v. What is the sum?

- vi. If you were to represent the sum using an arrow, how long would the arrow be, and what direction would it point?
- vii. Do you think that adding two negative numbers will always give you a smaller negative number? Why?
- c. What do both examples have in common?

RULE: Add rational numbers with the same sign by adding the absolute values and using the common sign.

Exercise 2

- a. Decide whether the sum will be positive or negative without actually calculating the sum.

i. $-4 + (-2)$

ii. $5 + 9$

iii. $-6 + (-3)$

iv. $-1 + (-11)$

v. $3 + 5 + 7$

vi. $-20 + (-15)$

b. Find the sum.

i. $15 + 7$

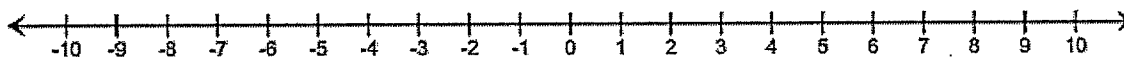
ii. $-4 + (-16)$

iii. $-18 + (-64)$

iv. $-205 + (-123)$

Example 2: Rule for Adding Opposite Signs

a. Represent $5 + (-3)$ using arrows on the number line.



i. How long is the arrow that represents 5?

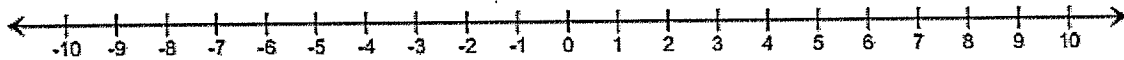
ii. What direction does it point?

iii. How long is the arrow that represents -3 ?

iv. What direction does it point?

- v. Which arrow is longer?
- vi. What is the sum? If you were to represent the sum using an arrow, how long would the arrow be, and what direction would it point?

b. Represent the $4 + (-7)$ using arrows on the number line.



- i. In the two examples above, what is the relationship between the length of the arrow representing the sum and the lengths of the arrows representing the two addends?
- ii. What is the relationship between the direction of the arrow representing the sum and the direction of the arrows representing the two addends?
- iii. Write a rule that will give the length and direction of the arrow representing the sum of two values that have opposite signs.

RULE: Add rational numbers with opposite signs by subtracting the absolute values and using the sign of the integer with the greater absolute value.

Exercise 3

- a. Circle the integer with the greater absolute value. Decide whether the sum will be positive or negative without actually calculating the sum.

i. $-1 + 2$

ii. $5 + (-9)$

iii. $-6 + 3$

iv. $-11 + 1$

- b. Find the sum.

i. $-10 + 7$

ii. $8 + (-16)$

iii. $-12 + (65)$

iv. $105 + (-126)$

Example 3: Applying Integer Addition Rules to Rational Numbers

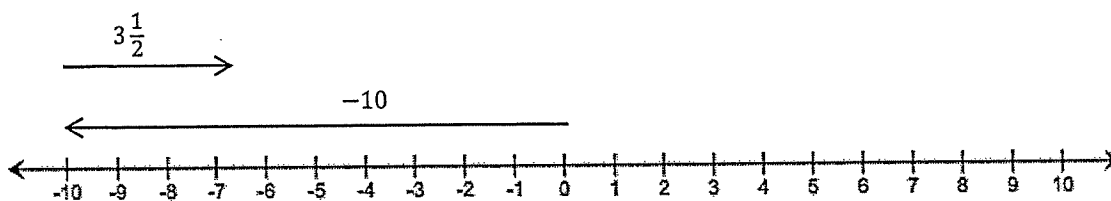
Find the sum of $6 + \left(-2\frac{1}{4}\right)$. The addition of rational numbers follows the same rules of addition for integers.

- Find the absolute values of the numbers.
- Subtract the absolute values.
- The answer will take the sign of the number that has the greater absolute value.

Exercise 4

Solve the following problems. Show your work.

- Find the sum of $-18 + 7$.
- If the temperature outside was 73 degrees at 5:00 p.m., but it fell 19 degrees by 10:00 p.m., what is the temperature at 10:00 p.m.? Write an equation and solve.
- Write an addition sentence, and find the sum using the diagram below.



Name _____

Date _____

Lesson 4: Efficiently Adding Integers and Other Rational Numbers

Exit Ticket

- Write an addition problem that has a sum of $-4\frac{3}{5}$ and
 - The two addends have the same sign.

 - The two addends have different signs.

- In the Integer Game, what card would you need to draw to get a score of 0 if you have a -16 , -35 , and 18 in your hand?

Name _____

Date _____

Lesson 4: Efficiently Adding Integers and Other Rational Numbers

Lesson Summary

- Add integers with the same sign by adding the absolute values and using the common sign.
- Steps to adding integers with opposite signs:
 1. Find the absolute values of the integers.
 2. Subtract the absolute values.
 3. The answer will take the sign of the integer that has the greater absolute value.
- To add rational numbers, follow the same rules used to add integers.

Problem Set

1. Find the sum. Show your work to justify your answer.

a. $4 + 17$

b. $-6 + (-12)$

c. $2.2 + (-3.7)$

d. $-3 + (-5) + 8$

e. $\frac{1}{3} + \left(-2\frac{1}{4}\right)$

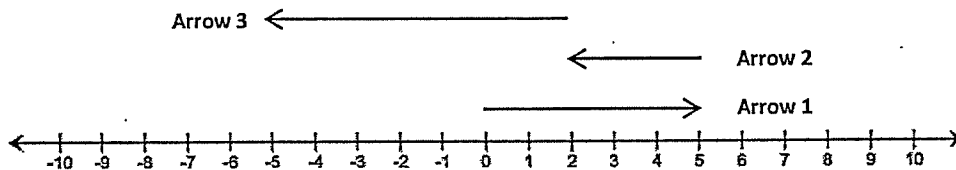
2. Which of these story problems describes the sum $19 + (-12)$? Check all that apply. Show your work to justify your answer.

_____ Jared's dad paid him \$19 for raking the leaves from the yard on Wednesday. Jared spent \$12 at the movie theater on Friday. How much money does Jared have left?

_____ Jared owed his brother \$19 for raking the leaves while Jared was sick. Jared's dad gave him \$12 for doing his chores for the week. How much money does Jared have now?

_____ Jared's grandmother gave him \$19 for his birthday. He bought \$8 worth of candy and spent another \$4 on a new comic book. How much money does Jared have left over?

3. Use the diagram below to complete each part.

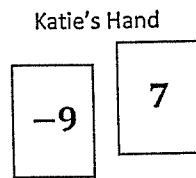
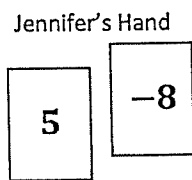


- a. Label each arrow with the number the arrow represents.
- b. How long is each arrow? What direction does each arrow point?

Arrow	Length	Direction
1		
2		
3		

- c. Write an equation that represents the sum of the numbers. Find the sum.

4. Jennifer and Katie were playing the Integer Game in class. Their hands are represented below.



- a. What is the value of each of their hands? Show your work to support your answer.
- b. If Jennifer drew two more cards, is it possible for the value of her hand not to change? Explain why or why not.
- c. If Katie wanted to win the game by getting a score of 0, what card would she need? Explain.
- d. If Jennifer drew -1 and -2 , what would be her new score? Show your work to support your answer.

Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers

Classwork

Example 1: Exploring Subtraction with the Integer Game

Play the Integer Game in your group. Start Round 1 by selecting four cards. Follow the steps for each round of play.

1. Write the value of your hand in the Total column.
2. Then, record what card values you select in the Action 1 column and discard from your hand in the Action 2 column.
3. After each action, calculate your new total, and record it under the appropriate Results column.
4. Based on the results, describe what happens to the value of your hand under the appropriate Descriptions column.
For example, "Score increased by 3."

Round	Total	Action 1	Result 1	Description	Action 2	Result 2	Description
1							
2							
3							
4							
5							

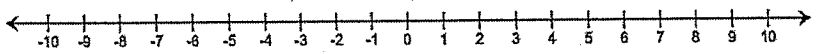
Example 2: Subtracting a Positive Number

Follow along with your teacher to complete the diagrams below.

4

2

$4 + 2 = \square$

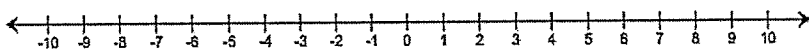


Show that discarding (subtracting) a positive card, which is the same as subtracting a positive number, decreases the value of the hand.

4

2

$4 + 2 - 2 = \square$



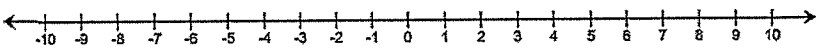
OR

4

2

-2

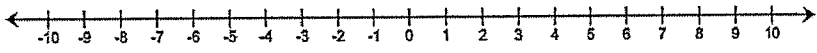
$4 + 2 + (-2) = \square$



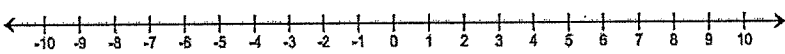
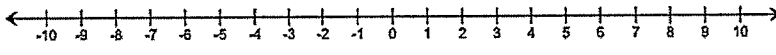
Removing () a positive card changes the score in the same way as a card whose value is the (or opposite). In this case, adding the corresponding

Example 3: Subtracting a Negative Number

Follow along with your teacher to complete the diagrams below.

4	-2	
$4 + (-2) = \square$		

How does removing a negative card change the score, or value, of the hand?

4	-2	
$4 + (-2) - (-2) = \square$		
OR		
4	-2	2
$4 + (-2) + 2 = \square$		

Removing () a negative card changes the score in the same way as a card whose value is the () (or opposite). In this case, adding the corresponding ().

THE RULE OF SUBTRACTION: *Subtracting a number is the same as adding its additive inverse (or opposite).*

Exercises 1–3: Subtracting Positive and Negative Integers

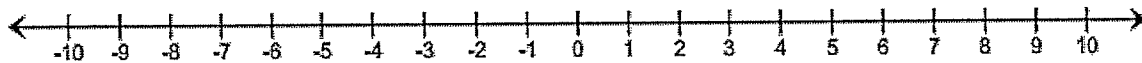
1. Using the rule of subtraction, rewrite the following subtraction sentences as addition sentences and solve. Use the number line below if needed.

a. $8 - 2$

b. $4 - 9$

c. $-3 - 7$

d. $-9 - (-2)$



2. Find the differences.

a. $-2 - (-5)$

b. $11 - (-8)$

c. $-10 - (-4)$

3. Write two equivalent expressions that represent the situation. What is the difference in their elevations?
An airplane flies at an altitude of 25,000 feet. A submarine dives to a depth of 600 feet below sea level.

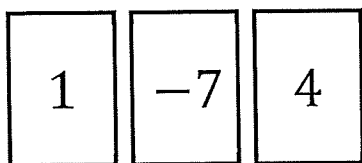
Name _____

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Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers

Exit Ticket

1. If a player had the following cards, what is the value of his hand?



- a. Identify two different ways the player could get to a score of 5 by adding or removing only one card. Explain.
- b. Write two equations for part (a), one for each of the methods you came up with for arriving at a score of 5.
2. Using the rule of subtraction, rewrite the following subtraction expressions as addition expressions, and find the sums.
- a. $5 - 9$
- b. $-14 - (-2)$

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Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers

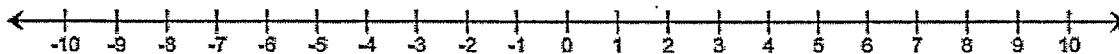
Lesson Summary

- **THE RULE OF SUBTRACTION:** Subtracting a number is the same as adding its opposite.
- Removing (subtracting) a positive card changes the score in the same way as adding a corresponding negative card.
- Removing (subtracting) a negative card makes the same change as adding the corresponding positive card.
- For all rational numbers, subtracting a number and adding it back gets you back to where you started:
 $(m - n) + n = m$.

Problem Set

1. On a number line, find the difference of each number and 4. Complete the table to support your answers. The first example is provided.

Number	Subtraction Expression	Addition Expression	Answer
10	$10 - 4$	$10 + (-4)$	6
2			
-4			
-6			
1			



3. Write the following expressions as a single integer.

a. $-2 + 16$

b. $-2 - (-16)$

c. $18 - 26$

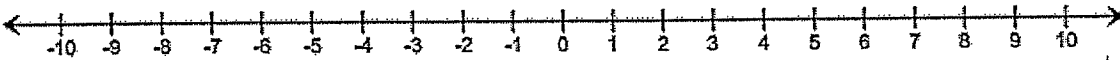
d. $-14 - 23$

e. $30 - (-45)$

4. Explain what is meant by the following, and illustrate with an example:

"For any real numbers, p and q , $p - q = p + (-q)$."

5. Choose an integer between -1 and -5 on the number line, and label it point P . Locate and label the following points on the number line. Show your work.



- a. Point $A: P - 5$
- b. Point $B: (P - 4) + 4$
- c. Point $C: -P - (-7)$

Challenge Problem:

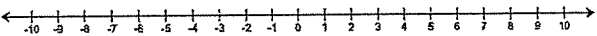
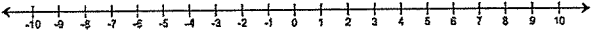
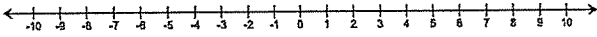
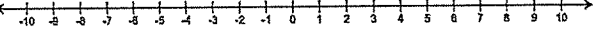
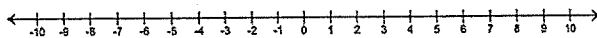
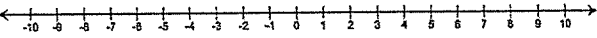
6. Write two equivalent expressions that represent the situation. What is the difference in their elevations?
An airplane flies at an altitude of 26,000 feet. A submarine dives to a depth of 700 feet below sea level.

Lesson 6: The Distance Between Two Rational Numbers

Classwork

Exercise 1

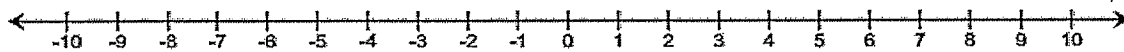
Use the number line to answer each of the following.

Person A	Person B
What is the distance between -4 and 5 ? 	What is the distance between 5 and -4 ? 
What is the distance between -5 and -3 ? 	What is the distance between -3 and -5 ? 
What is the distance between 7 and -1 ? 	What is the distance between -1 and 7 ? 

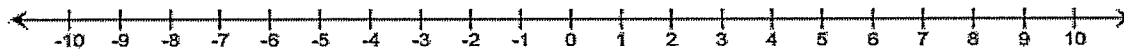
Exercise 2

Use the number line to answer each of the following questions.

- a. What is the distance between 0 and -8 ?



- b. What is the distance between -2 and $-1\frac{1}{2}$?



- c. What is the distance between -6 and -10 ?

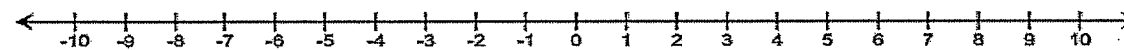


Example 1: Formula for the Distance Between Two Rational Numbers

Find the distance between -3 and 2 .

Step 1: Start on an endpoint.

Step 2: Count the number of units from the endpoint you started on to the other endpoint.



Using a formula, _____

For two rational numbers p and q , the distance between p and q is $|p - q|$.

- c. Would your answer for part (b) be different if the temperature dropped from 15°F to -18°F ? Explain.
- d. Beryl is the first person to finish a 5K race and is standing 15 feet beyond the finish line. Another runner, Jeremy, is currently trying to finish the race and has approximately 14 feet before he reaches the finish line. What is the minimum possible distance between Beryl and Jeremy?
- e. What is the change in elevation from 140 feet above sea level to 40 feet below sea level? Explain.

Name _____

Date _____

Lesson 6: The Distance Between Two Rational Numbers

Exit Ticket

Two Grade 7 students, Monique and Matt, both solved the following math problem:

If the temperature drops from 7°F to -17°F , by how much did the temperature *decrease*?

The students came up with different answers. Monique said the answer is 24°F , and Matt said the answer is 10°F . Who is correct? Explain, and support your written response with the use of a formula and a vertical number line diagram.

Name _____

Date _____

Lesson 6: The Distance Between Two Rational Numbers

Lesson Summary

- To find the distance between two rational numbers on a number line, you can count the number of units between the numbers.
- Using a formula, the distance between rational numbers, p and q , is $|p - q|$.
- Distance is always positive.
- Change may be positive or negative. For instance, there is a -4° change when the temperature goes from 7° to 3° .

Problem Set

1. $|-19 - 12|$

2. $|19 - (-12)|$

3. $|10 - (-43)|$

4. $|-10 - 43|$

5. $|-1 - (-16)|$

6. $|1 - 16|$

7. $|0 - (-9)|$

8. $|0 - 9|$

9. $|-14.5 - 13|$

10. $|14.5 - (-13)|$

11. Describe any patterns you see in the answers to the problems in the left- and right-hand columns. Why do you think this pattern exists?

Lesson 7: Addition and Subtraction of Rational Numbers

Classwork

Exercise 1: Real-World Connection to Adding and Subtracting Rational Numbers

Suppose a seventh grader's birthday is today, and she is 12 years old. How old was she $3\frac{1}{2}$ years ago? Write an equation, and use a number line to model your answer.

Example 1: Representing Sums of Rational Numbers on a Number Line

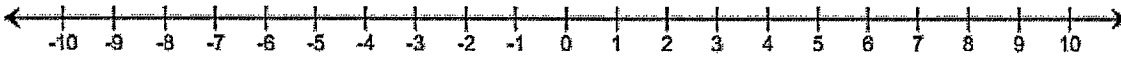
- Place the tail of the arrow on 12.
- The length of the arrow is the absolute value of $-3\frac{1}{2}$, $|-3\frac{1}{2}| = 3\frac{1}{2}$.
- The direction of the arrow is to the *left* since you are adding a negative number to 12.

Draw the number line model in the space below.



Exercise 2

Find the following sum using a number line diagram: $-2\frac{1}{2} + 5$.

**Example 2: Representing Differences of Rational Numbers on a Number Line**

Find the following difference, and represent it on a number line: $1 - 2\frac{1}{4}$.

a.

Now follow the steps to represent the sum:

b.

c.

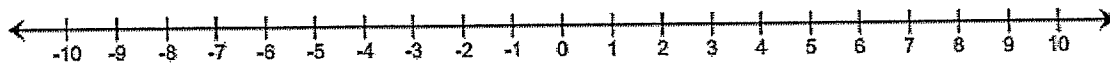
d.

Draw the number line model in the space below.



Exercise 3

Find the following difference, and represent it on a number line: $-5\frac{1}{2} - (-8)$.



Exercise 4

Find the following sums and differences using a number line model.

a. $-6 + 5\frac{1}{4}$

b. $7 - (-0.9)$

c. $2.5 + \left(-\frac{1}{2}\right)$

d. $-\frac{1}{4} + 4$

Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

Classwork

Example 1: The Opposite of a Sum is the Sum of its Opposites

Explain the meaning of “The opposite of a sum is the sum of its opposites.” Use a specific math example.

Rational Number	Rational Number	Sum	Opposite of the Sum

Opposite Rational Number	Opposite Rational Number	Sum

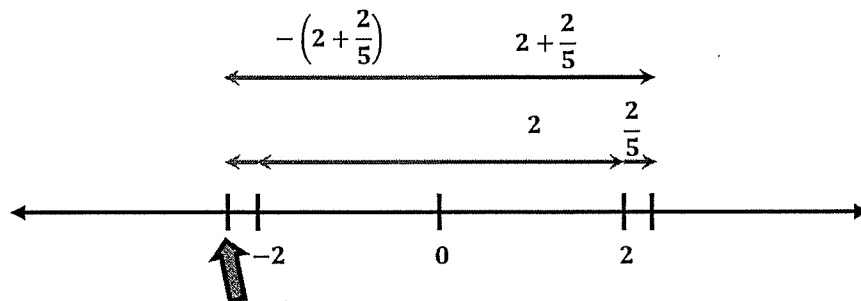
Exercise 1

Represent the following expression with a single rational number.

$$-2\frac{2}{5} + 3\frac{1}{4} - \frac{3}{5}$$

Example 2: A Mixed Number Is a Sum

Use the number line model shown below to explain and write the opposite of $2\frac{2}{5}$ as a sum of two rational numbers.



The opposite of a sum (top single arrow pointing left) and the sum of the opposites correspond to the same point on the number line.

Exercise 2

Rewrite each mixed number as the sum of two signed numbers.

a. $-9\frac{5}{8}$

b. $-2\frac{1}{2}$

c. $8\frac{11}{12}$

Exercise 3

Represent each sum as a mixed number.

a. $-1 + \left(-\frac{5}{12}\right)$

b. $30 + \frac{1}{8}$

c. $-17 + \left(-\frac{1}{9}\right)$

Exercise 4

Mr. Mitchell lost 10 pounds over the summer by jogging each week. By winter, he had gained $5\frac{1}{8}$ pounds. Represent this situation with an expression involving signed numbers. What is the overall change in Mr. Mitchell's weight?

Exercise 5

Jamal is completing a math problem and represents the expression $-5\frac{5}{7} + 8 - 3\frac{2}{7}$ with a single rational number as shown in the steps below. Justify each of Jamal's steps. Then, show another way to solve the problem.

$$\begin{aligned} &= -5\frac{5}{7} + 8 + \left(-3\frac{2}{7}\right) \\ &= -5\frac{5}{7} + \left(-3\frac{2}{7}\right) + 8 \\ &= -5 + \left(-\frac{5}{7}\right) + (-3) + \left(-\frac{2}{7}\right) + 8 \\ &= -5 + \left(-\frac{5}{7}\right) + \left(-\frac{2}{7}\right) + (-3) + 8 \\ &= -5 + (-1) + (-3) + 8 \\ &= -6 + (-3) + 8 \\ &= (-9) + 8 \\ &= -1 \end{aligned}$$

Name _____

Date _____

Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

Exit Ticket

Mariah and Shane both started to work on a math problem and were comparing their work in math class. Are both of their representations correct? Explain, and finish the math problem correctly to arrive at the correct answer.

Math Problem

Jessica's friend lent her \$5. Later that day Jessica gave her friend back $1\frac{3}{4}$ dollars.

Which rational number represents the overall change to the amount of money Jessica's friend has?

Mariah started the problem as follows:

$$\begin{aligned} -5 - \left(-1\frac{3}{4}\right) \\ = -5 + 1 - \frac{3}{4} \end{aligned}$$

Shane started the problem as follows:

$$\begin{aligned} -5 - \left(-1\frac{3}{4}\right) \\ = -5 + \left(1\frac{3}{4}\right) \\ = -5 + \left(1 + \frac{3}{4}\right) \end{aligned}$$

Name _____

Date _____

Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,

$$-5\frac{2}{9} + 3.7 + 5\frac{2}{9} = \left(-5\frac{2}{9} + 5\frac{2}{9}\right) + 3.7 = 0 + 3.7 = 3.7.$$

- The opposite of a sum is the sum of its opposites as shown in the examples that follow:

$$-4\frac{4}{7} = -4 + \left(-\frac{4}{7}\right)$$

$$-(5 + 3) = -5 + (-3)$$

Problem Set

- Represent each sum as a single rational number.

a. $-14 + \left(-\frac{8}{9}\right)$

b. $7 + \frac{1}{9}$

c. $-3 + \left(-\frac{1}{6}\right)$

Rewrite each of the following to show that *the opposite of a sum is the sum of the opposites*. Problem 2 has been completed as an example.

2. $-(9 + 8) = -9 + (-8)$
 $-17 = -17$

3. $-\left(\frac{1}{4} + 6\right)$

4. $-(10 + (-6))$

5. $-\left((-55) + \frac{1}{2}\right)$

9. A number added to its opposite equals zero. What do you suppose is true about *a sum added to its opposite*?
Use the following examples to reach a conclusion. Express the answer to each example as a single rational number.

a. $(3 + 4) + (-3 + -4)$

b. $(-8 + 1) + (8 + (-1))$

c. $\left(-\frac{1}{2} + \left(-\frac{1}{4}\right)\right) + \left(\frac{1}{2} + \frac{1}{4}\right)$

Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

Classwork

Exercise 1

Unscramble the cards, and show the steps in the correct order to arrive at the solution to $5\frac{2}{9} - (8.1 + 5\frac{2}{9})$.

$$0 + (-8.1)$$

$$\left(5\frac{2}{9} + \left(-5\frac{2}{9}\right)\right) + (-8.1)$$

$$-8.1$$

$$5\frac{2}{9} + \left(-8.1 + \left(-5\frac{2}{9}\right)\right)$$

$$5\frac{2}{9} + \left(-5\frac{2}{9} + (-8.1)\right)$$

Examples 1–2

Represent each of the following expressions as one rational number. Show and explain your steps.

1. $4\frac{4}{7} - \left(4\frac{4}{7} - 10\right)$

2. $5 + \left(-4\frac{4}{7}\right)$

Exercise 2: Team Work!

a. $-5.2 - (-3.1) + 5.2$

b. $32 + \left(-12\frac{7}{8}\right)$

c. $3\frac{1}{6} + 20.3 - \left(-5\frac{5}{6}\right)$

d. $\frac{16}{20} - (-1.8) - \frac{4}{5}$

Exercise 3

Explain, step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

$$-24 - \left(-\frac{1}{2}\right) - 12.5$$

Name _____

Date _____

Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

Exit Ticket

1. Jamie was working on his math homework with his friend, Kent. Jamie looked at the following problem.

$$-9.5 - (-8) - 6.5$$

He told Kent that he did not know how to subtract negative numbers. Kent said that he knew how to solve the problem using only addition. What did Kent mean by that? Explain. Then, show your work, and represent the answer as a single rational number.

Work Space:

Answer: _____

2. Use one rational number to represent the following expression. Show your work.

$$3 + (-0.2) - 15\frac{1}{4}$$

Name _____

Date _____

Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,

$$-5\frac{2}{9} + 3.7 + 5\frac{2}{9} = \left(-5\frac{2}{9} + 5\frac{2}{9}\right) + 3.7 = 0 + 3.7 = 3.7.$$

- The opposite of a sum is the sum of its opposites as shown in the examples that follow:

$$-4\frac{4}{7} = -4 + \left(-\frac{4}{7}\right).$$

$$-(5 + 3) = -5 + (-3).$$

Problem Set

Show all steps taken to rewrite each of the following as a single rational number.

1. $80 + \left(-22\frac{4}{15}\right)$

2. $10 + \left(-3\frac{3}{8}\right)$

3. $\frac{1}{5} + 20.3 - \left(-5\frac{3}{5}\right)$

4. $\frac{11}{12} - (-10) - \frac{5}{6}$

5. Explain, step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

$$1 - \frac{3}{4} + \left(-12\frac{1}{4}\right)$$

Lesson 10: Understanding Multiplication of Integers

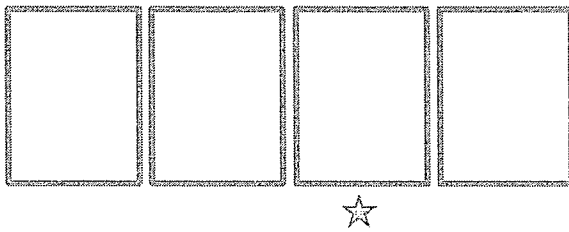
Classwork

Exercise 1: Integer Game Revisited

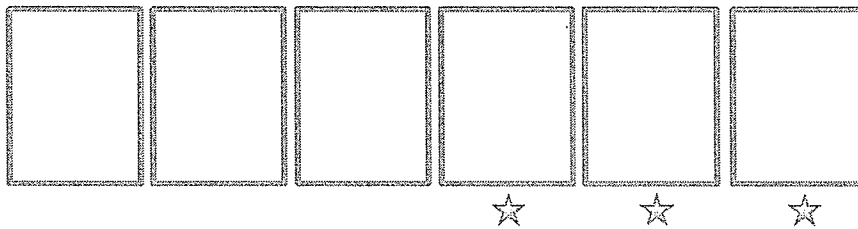
In groups of four, play one round of the Integer Game (see Integer Game outline for directions).

Example 1: Product of a Positive Integer and a Negative Integer

Part A:



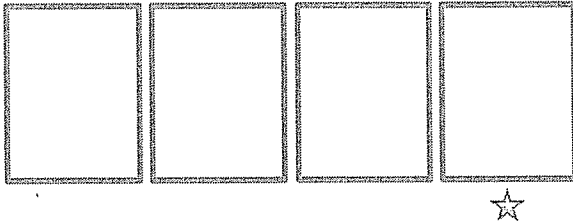
Part B:



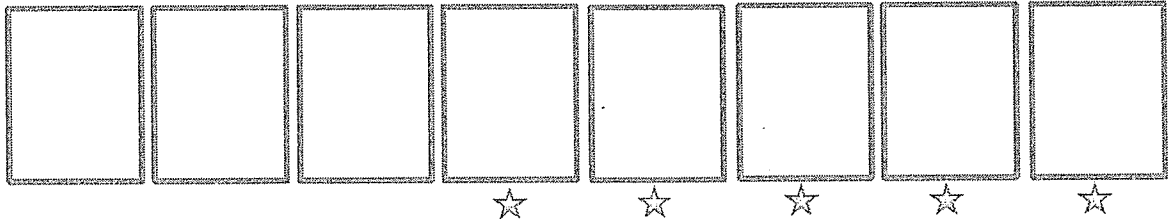
Use your cards from Part B to answer the questions below.

- Write a product that describes the three matching cards.
- Write an expression that represents how each of the \star cards changes your score.
- Write an equation that relates these two expressions.
- Write an integer that represents the total change to your score by the three \star cards.
- Write an equation that relates the product and how it affects your score.

Part C:



Part D:

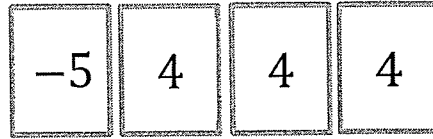


Use your cards from Part D to answer the questions below.

- f. Write a product that describes the five matching cards.
- g. Write an expression that represents how each of the ★ cards changes your score.
- h. Write an equation that relates these two expressions.
- i. Write an integer that represents the total change to your score by the five ★ cards.
- j. Write an equation that relates the product and how it affects your score.
- k. Use the expression 5×4 to relate the multiplication of a positive valued card to addition.
- l. Use the expression $3 \times (-5)$ to relate the multiplication of a negative valued card to addition.

Example 2: Product of a Negative Integer and a Positive Integer

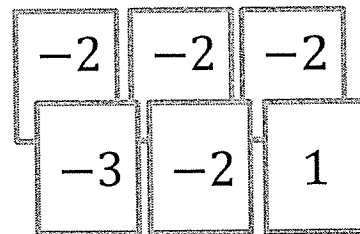
- a. If all of the 4's from the playing hand on the right are discarded, how will the score be affected? Model this using a product in an equation.



- b. What three matching cards could be added to those pictured to get the same change in score? Model this using a product in an equation.
- c. Seeing how each play affects the score, relate the products that you used to model them. What do you conclude about multiplying integers with opposite signs?

Example 3: Product of Two Negative Integers

- a. If the matching cards from the playing hand on the right are discarded, how will this hand's score be affected? Model this using a product in an equation.



- b. What four matching cards could be added to those pictured to get the same change in score? Model this using a product in an equation.

- c. Seeing how each play affects the score, relate the products that you used to model them. What do you conclude about multiplying integers with the same sign?
- d. Using the conclusions from Examples 2 and 3, what can we conclude about multiplying integers? Write a few examples.

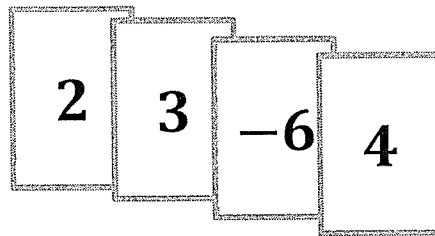
Name _____

Date _____

Lesson 10: Understanding Multiplication of Integers

Exit Ticket

1. Natalie is playing the Integer Game and only shows you the four cards shown below. She tells you that the rest of her cards have the same values on them and match one of these four cards.



- a. If all of the matching cards will increase her score by 18, what are the matching cards?
- b. If all of the matching cards will decrease her score by 12, what are the matching cards?
2. A hand of six integer cards has one matching set of two or more cards. If the matching set of cards is removed from the hand, the score of the hand will increase by six. What are the possible values of these matching cards? Explain. Write an equation using multiplication showing how the matching cards yield an increase in score of six.

Name _____

Date _____

Lesson 10: Understanding Multiplication of Integers

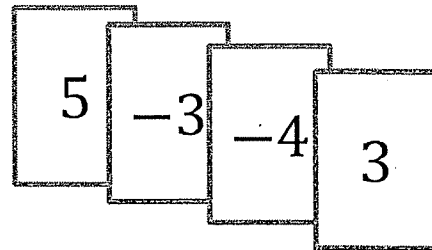
Lesson Summary

Multiplying integers is repeated addition and can be modeled with the Integer Game. If $3 \times a$ corresponds to what happens to your score if you get three cards of value a , then $(-3) \times a$ corresponds to what happens to your score if you lose three cards of value a . Adding a number multiple times has the same effect as removing the opposite value the same number of times (e.g., $a \times b = (-a) \times (-b)$ and $a \times (-b) = (-a) \times b$).

Problem Set

1. Describe sets of two or more matching integer cards that satisfy the criteria in each part below:
 - a. Cards increase the score by eight points.
 - b. Cards decrease the score by 9 points.
 - c. Removing cards that increase the score by 10 points.
 - d. Positive cards that decrease the score by 18 points.

2. You have the integer cards shown at the right when your teacher tells you to choose a card to multiply four times. If your goal is to get your score as close to zero as possible, which card would you choose? Explain how your choice changes your score.



3. Sherry is playing the Integer Game and is given a chance to discard a set of matching cards. Sherry determines that if she discards one set of cards, her score will increase by 12. If she discards another set, then her score will decrease by eight. If her matching cards make up all six cards in her hand, what cards are in Sherry's hand? Are there any other possibilities?

Lesson 11: Develop Rules for Multiplying Signed Numbers

Classwork

Example 1: Extending Whole Number Multiplication to the Integers

Part A: Complete quadrants *I* and *IV* of the table below to show how sets of matching integer cards will affect a player's score in the Integer Game. For example, three 2s would increase a player's score by $0 + 2 + 2 + 2 = 6$ points.

	Quadrant <i>II</i>						Quadrant <i>I</i>					
What does this quadrant represent? _____					5						What does this quadrant represent? _____	
					4							
					3							
					2			6				
					1							
						←	Number of matching cards					
What does this quadrant represent? _____							1	2	3	4	5	What does this quadrant represent? _____
					-1							
					-2							
					-3							
					-4							
				-5								
	Quadrant <i>III</i>					↑	Quadrant <i>IV</i>					
	Integer card values											

- a. What patterns do you see in the right half of the table?

- b. Enter the missing integers in the left side of the middle row, and describe what they represent.

Example 2: Using Properties of Arithmetic to Explain Multiplication of Negative Numbers**Exercise 1: Multiplication of Integers in the Real World**

Generate real-world situations that can be modeled by each of the following multiplication problems. Use the Integer Game as a resource.

a. -3×5

b. $-6 \times (-3)$

c. $4 \times (-7)$

Name _____

Date _____

Lesson 11: Develop Rules for Multiplying Signed Numbers

Lesson Summary

To multiply signed numbers, multiply the absolute values to get the absolute value of the product. The sign of the product is positive if the factors have the same sign and negative if they have opposite signs.

Problem Set

1. Complete the problems below. Then, answer the question that follows.

$3 \times 3 =$	$3 \times 2 =$	$3 \times 1 =$	$3 \times 0 =$	$3 \times (-1) =$	$3 \times (-2) =$
$2 \times 3 =$	$2 \times 2 =$	$2 \times 1 =$	$2 \times 0 =$	$2 \times (-1) =$	$2 \times (-2) =$
$1 \times 3 =$	$1 \times 2 =$	$1 \times 1 =$	$1 \times 0 =$	$1 \times (-1) =$	$1 \times (-2) =$
$0 \times 3 =$	$0 \times 2 =$	$0 \times 1 =$	$0 \times 0 =$	$0 \times (-1) =$	$0 \times (-2) =$
$-1 \times 3 =$	$-1 \times 2 =$	$-1 \times 1 =$	$-1 \times 0 =$	$-1 \times (-1) =$	$-1 \times (-2) =$
$-2 \times 3 =$	$-2 \times 2 =$	$-2 \times 1 =$	$-2 \times 0 =$	$-2 \times (-1) =$	$-2 \times (-2) =$
$-3 \times 3 =$	$-3 \times 2 =$	$-3 \times 1 =$	$-3 \times 0 =$	$-3 \times (-1) =$	$-3 \times (-2) =$

Which row shows the same pattern as the outlined column? Are the problems similar or different? Explain.

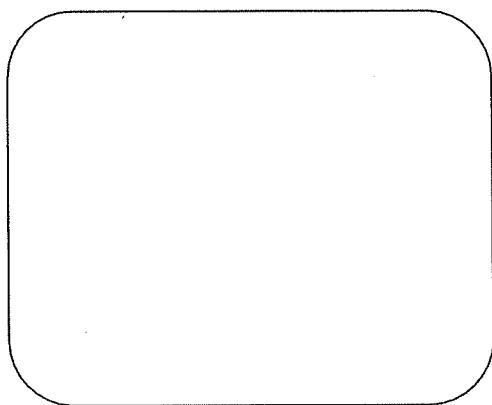
Lesson 12: Division of Integers

Classwork

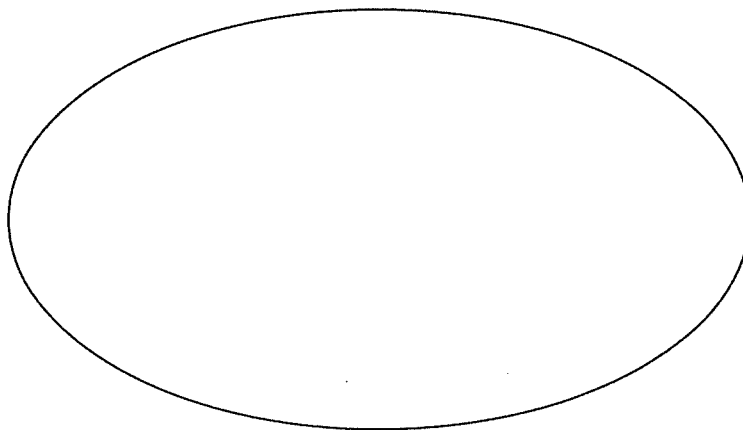
Exercise 1: Recalling the Relationship Between Multiplication and Division

Record equations from Exercise 1 on the left.

Equations



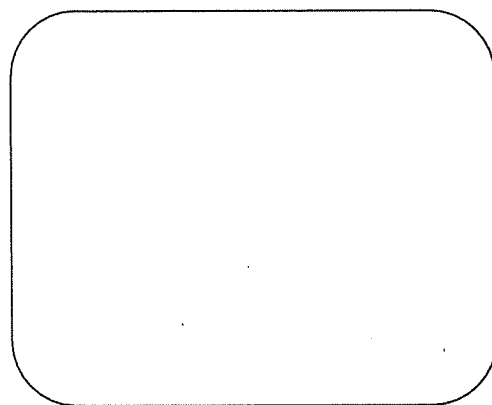
Integers



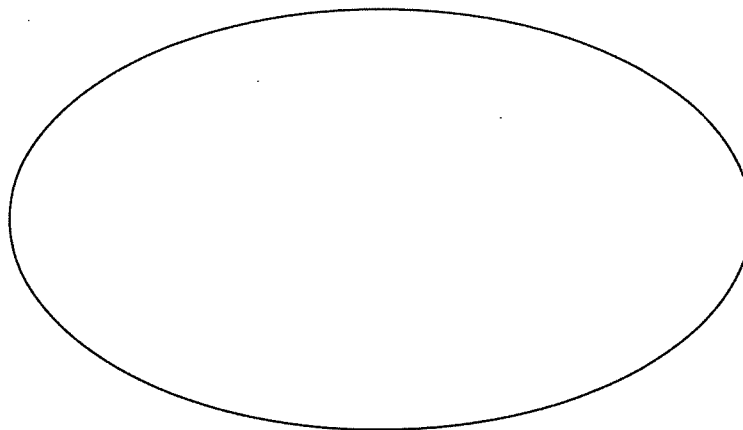
Example 1: Transitioning from Integer Multiplication Rules to Integer Division Rules

Record your group's number sentences in the space on the left below.

Equations



Integers



- a. List examples of division problems that produced a quotient that is a negative number.
- b. If the quotient is a negative number, what must be true about the signs of the dividend and divisor?
- c. List your examples of division problems that produced a quotient that is a positive number.
- d. If the quotient is a positive number, what must be true about the signs of the dividend and divisor?

Rules for Dividing Two Integers:

- A quotient is negative if the divisor and the dividend have _____ signs.
- A quotient is positive if the divisor and the dividend have _____ signs.

Exercise 2: Is the Quotient of Two Integers Always an Integer?

Is the quotient of two integers always an integer? Use the work space below to create quotients of integers. Answer the question, and use examples or a counterexample to support your claim.

Work Space:

Answer:

Exercise 3: Different Representation of the Same Quotient

Are the answers to the three quotients below the same or different? Why or why not?

a. $-14 \div 7$

b. $14 \div (-7)$

c. $-(14 \div 7)$

Name _____

Date _____

Lesson 12: Division of Integers

Exit Ticket

1. Mrs. McIntire, a seventh-grade math teacher, is grading papers. Three students gave the following responses to the same math problem:

Student one: $\frac{1}{-2}$

Student two: $-\left(\frac{1}{2}\right)$

Student three: $-\frac{1}{2}$

On Mrs. McIntire's answer key for the assignment, the correct answer is -0.5 . Which student answer(s) is (are) correct? Explain.

2. Complete the table below. Provide an answer for each integer division problem, and write a related equation using integer multiplication.

Integer Division Problem	Related Equation Using Integer Multiplication
$-36 \div (-9) = \underline{\hspace{2cm}}$	
$24 \div (-8) = \underline{\hspace{2cm}}$	
$50 \div 10 = \underline{\hspace{2cm}}$	
$42 \div 6 = \underline{\hspace{2cm}}$	

Name _____

Date _____

Lesson 12: Division of Integers

Lesson Summary

The rules for dividing integers are similar to the rules for multiplying integers (when the divisor is not zero). The quotient is positive if the divisor and dividend have the same signs and negative if they have opposite signs.

The quotient of any two integers (with a nonzero divisor) will be a rational number. If p and q are integers, then

$$-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$$

Problem Set

1. Find the missing values in each column.

Column A	Column B	Column C	Column D
$48 \div 4 =$	$24 \div 4 =$	$63 \div 7 =$	$21 \div 7 =$
$-48 \div (-4) =$	$-24 \div (-4) =$	$-63 \div (-7) =$	$-21 \div (-7) =$
$-48 \div 4 =$	$-24 \div 4 =$	$-63 \div 7 =$	$-21 \div 7 =$
$48 \div (-4) =$	$24 \div (-4) =$	$63 \div (-7) =$	$21 \div (-7) =$

2. Describe the pattern you see in each column's answers in Problem 1, relating it to the problems' divisors and dividends. Why is this so?

3. Describe the pattern you see between the answers for Columns A and B in Problem 1 (e.g., compare the first answer in Column A to the first answer in Column B; compare the second answer in Column A to the second answer in Column B). Why is this so?

4. Describe the pattern you see between the answers for Columns C and D in Problem 1. Why is this so?

Lesson 13: Converting Between Fractions and Decimals Using Equivalent Fractions

Classwork

Example 1: Representations of Rational Numbers in the Real World

Following the Opening Exercise and class discussion, describe why we need to know how to represent rational numbers in different ways.

Example 2: Using Place Values to Write (Terminating) Decimals as Equivalent Fractions

- What is the value of the number 2.25? How can this number be written as a fraction or mixed number?
- Rewrite the fraction in its simplest form showing all steps that you use.
- What is the value of the number 2.025? How can this number be written as a mixed number?
- Rewrite the fraction in its simplest form showing all steps that you use.

Exercise 1

Use place value to convert each terminating decimal to a fraction. Then rewrite each fraction in its simplest form.

a. 0.218

b. 0.16

c. 2.72

d. 0.0005

Example 3: Converting Fractions to Decimals—Fractions with Denominators Having Factors of only 2 or 5

- a. What are
- decimals*
- ?

- b. Use the meaning of *decimal* to relate decimal place values.
- c. Write the number $\frac{3}{100}$ as a decimal. Describe your process.
- d. Write the number $\frac{3}{20}$ as a decimal. Describe your process.
- e. Write the number $\frac{10}{25}$ as a decimal. Describe your process.
- f. Write the number $\frac{8}{40}$ as a decimal. Describe your process.

Exercise 2

Convert each fraction to a decimal using an equivalent fraction.

a. $\frac{3}{16} =$

b. $\frac{7}{5} =$

c. $\frac{11}{32} =$

d. $\frac{35}{50} =$

Name _____

Date _____

Lesson 13: Converting Between Fractions and Decimals Using Equivalent Fractions

Lesson Summary

Any terminating decimal can be converted to a fraction using place value (e.g., 0.35 is thirty-five hundredths or $\frac{35}{100}$). A fraction whose denominator includes only factors of 2 and 5 can be converted to a decimal by writing the denominator as a power of ten.

Problem Set

1. Convert each terminating decimal to a fraction in its simplest form.

a. 0.4

b. 0.16

c. 0.625

d. 0.08

e. 0.012

2. Convert each fraction or mixed number to a decimal using an equivalent fraction.

a. $\frac{4}{5}$

b. $\frac{3}{40}$

c. $\frac{8}{200}$

d. $3\frac{5}{16}$

3. Tanja is converting a fraction into a decimal by finding an equivalent fraction that has a power of 10 in the denominator. Sara looks at the last step in Tanja's work (shown below) and says that she cannot go any further. Is Sara correct? If she is, explain why. If Sara is incorrect, complete the remaining steps.

$$\frac{72}{480} = \frac{2^3 \cdot 3^2}{2^5 \cdot 3 \cdot 5}$$

Example 3: Converting Rational Numbers to Decimals Using Long Division

Use the long division algorithm to find the decimal value of $-\frac{3}{4}$.

Exercise 1

Convert each rational number to its decimal form using long division.

a. $-\frac{7}{8} =$

b. $\frac{3}{16} =$

Example 4: Converting Rational Numbers to Decimals Using Long Division

Use long division to find the decimal representation of $\frac{1}{3}$.

Exercise 2

Calculate the decimal values of the fraction below using long division. Express your answers using bars over the shortest sequence of repeating digits.

a. $-\frac{4}{9}$

b. $-\frac{1}{11}$

c. $\frac{1}{7}$

d. $-\frac{5}{6}$

Name _____

Date _____

Lesson 14: Converting Rational Numbers to Decimals Using Long Division

Division

Exit Ticket

1. What is the decimal value of $\frac{4}{11}$?

a.

b.

c.

2. How do you know that $\frac{4}{11}$ is a repeating decimal?

3. What causes a repeating decimal in the long division algorithm?

Name _____

Date _____

Lesson 14: Converting Rational Numbers to Decimals Using Long Division

Division

Lesson Summary

The real world requires that we represent rational numbers in different ways depending on the context of a situation. All rational numbers can be represented as either terminating decimals or repeating decimals using the long division algorithm. We represent repeating decimals by placing a bar over the shortest sequence of repeating digits.

Problem Set

1. Convert each rational number into its decimal form.

$$\frac{1}{3} = \underline{\hspace{2cm}}$$

$$\frac{1}{6} = \underline{\hspace{2cm}}$$

$$\frac{2}{6} = \underline{\hspace{2cm}}$$

$$\frac{3}{6} = \underline{\hspace{2cm}}$$

$$\frac{2}{3} = \underline{\hspace{2cm}}$$

$$\frac{4}{6} = \underline{\hspace{2cm}}$$

$$\frac{5}{6} = \underline{\hspace{2cm}}$$

$$\frac{1}{9} = \underline{\hspace{2cm}}$$

$$\frac{2}{9} = \underline{\hspace{2cm}}$$

$$\frac{3}{9} = \underline{\hspace{2cm}}$$

$$\frac{4}{9} = \underline{\hspace{2cm}}$$

$$\frac{5}{9} = \underline{\hspace{2cm}}$$

$$\frac{6}{9} = \underline{\hspace{2cm}}$$

$$\frac{7}{9} = \underline{\hspace{2cm}}$$

$$\frac{8}{9} = \underline{\hspace{2cm}}$$

One of these decimal representations is not like the others. Why?

Enrichment:

2. Chandler tells Aubrey that the decimal value of $-\frac{1}{17}$ is not a repeating decimal. Should Aubrey believe him? Explain.

3. Complete the quotients below without using a calculator, and answer the questions that follow.
- a. Convert each rational number in the table to its decimal equivalent.

$\frac{1}{11} =$	$\frac{2}{11} =$	$\frac{3}{11} =$	$\frac{4}{11} =$	$\frac{5}{11} =$
$\frac{6}{11} =$	$\frac{7}{11} =$	$\frac{8}{11} =$	$\frac{9}{11} =$	$\frac{10}{11} =$

Do you see a pattern? Explain.

- b. Convert each rational number in the table to its decimal equivalent.

$\frac{0}{99} =$	$\frac{10}{99} =$	$\frac{20}{99} =$	$\frac{30}{99} =$	$\frac{45}{99} =$
$\frac{58}{99} =$	$\frac{62}{99} =$	$\frac{77}{99} =$	$\frac{81}{99} =$	$\frac{98}{99} =$

Do you see a pattern? Explain.

- c. Can you find other rational numbers that follow similar patterns?

Lesson 15: Multiplication and Division of Rational Numbers

Classwork

Exercise 1

- a. In the space below, create a word problem that involves integer multiplication. Write an equation to model the situation.

- b. Now change the word problem by replacing the integers with non-integer rational numbers (fractions or decimals), and write the new equation.

- c. Was the process used to solve the second problem different from the process used to solve the first? Explain.

- d. The Rules for Multiplying Rational Numbers are the same as the Rules for Multiplying Integers:

1. _____
2. _____
3. _____

Exercise 2

- a. In one year, Melinda's parents spend \$2,640.90 on cable and internet service. If they spend the same amount each month, what is the resulting monthly change in the family's income?

- b. The Rules for Dividing Rational Numbers are the same as the Rules for Dividing Integers:

1. _____
2. _____
3. _____

Exercise 3

Use the fundraiser chart to help answer the questions that follow.

Grimes Middle School Flower Fundraiser

Customer	Plant Type	Number of Plants	Price per Plant	Total	Paid? Yes or No
Tamara Jones	tulip	2	\$4.25		No
Mrs. Wolff	daisy	1	\$3.75	\$ 3.75	Yes
Mr. Clark	geranium	5	\$2.25		Yes
Susie (Jeremy's sister)	violet	1	\$2.50	\$ 2.50	Yes
Nana and Pop (Jeremy's grandparents)	daisy	4	\$3.75	\$15.00	No

Jeremy is selling plants for the school's fundraiser, and listed above is a chart from his fundraiser order form. Use the information in the chart to answer the following questions. Show your work, and represent the answer as a rational number; then, explain your answer in the context of the situation.

- a. If Tamara Jones writes a check to pay for the plants, what is the resulting change in her checking account balance?

Numerical Answer:

Explanation:

- b. Mr. Clark wants to pay for his order with a \$20 bill, but Jeremy does not have change. Jeremy tells Mr. Clark he will give him the change later. How will this affect the total amount of money Jeremy collects? Explain. What rational number represents the change that must be made to the money Jeremy collects?

Numerical Answer:

Explanation:

- c. Jeremy's sister, Susie, borrowed the money from their mom to pay for her order. Their mother has agreed to deduct an equal amount of money from Susie's allowance each week for the next five weeks to repay the loan. What is the weekly change in Susie's allowance?

Numerical Answer:

Explanation:

- d. Jeremy's grandparents want to change their order. They want to order three daisies and one geranium, instead of four daisies. How does this change affect the amount of their order? Explain how you arrived at your answer.
- e. Jeremy approaches three people who do not want to buy any plants; however, they wish to donate some money for the fundraiser when Jeremy delivers the plants one week later. If the people promise to donate a total of \$14.40, what will be the average cash donation?
- f. Jeremy spends one week collecting orders. If 22 people purchase plants totaling \$270, what is the average amount of Jeremy's sale?

Name _____

Date _____

Lesson 15: Multiplication and Division of Rational Numbers

Exit Ticket

Harrison made up a game for his math project. It is similar to the Integer Game; however, in addition to integers, there are cards that contain other rational numbers such as -0.5 and -0.25 . Write a multiplication or division equation to represent each problem below. Show all related work.

1. Harrison discards three -0.25 cards from his hand. How does this affect the overall point value of his hand? Write an equation to model this situation.

1
2
3

2. Ezra and Benji are playing the game with Harrison. After Ezra doubles his hand's value, he has a total of -14.5 points. What was his hand's value before he doubled it?

3. Benji has four -0.5 cards. What is his total score?

Name _____

Date _____

Lesson 15: Multiplication and Division of Rational Numbers

Lesson Summary

The rules that apply for multiplying and dividing integers apply to rational numbers. We can use the products and quotients of rational numbers to describe real-world situations.

Problem Set

1. At lunch time, Benjamin often borrows money from his friends to buy snacks in the school cafeteria. Benjamin borrowed \$0.75 from his friend Clyde five days last week to buy ice cream bars. Represent the amount Benjamin borrowed as the product of two rational numbers; then, determine how much Benjamin owed his friend last week.
2. Monica regularly records her favorite television show. Each episode of the show requires 3.5% of the total capacity of her video recorder. Her recorder currently has 62% of its total memory free. If Monica records all five episodes this week, how much space will be left on her video recorder?

For Problems 3–5, find at least two possible sets of values that will work for each problem.

3. Fill in the blanks with two rational numbers (other than 1 and -1). $\underline{\hspace{1cm}} \times \left(-\frac{1}{2}\right) \times \underline{\hspace{1cm}} = -20$
What must be true about the relationship between the two numbers you chose?

4. Fill in the blanks with two rational numbers (other than 1 and -1). $-5.6 \times 100 \div 80 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 700$
What must be true about the relationship between the two numbers you chose?

5. Fill in the blanks with two rational numbers. $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = -0.75$
What must be true about the relationship between the two numbers you chose?

For Problems 6–8, create word problems that can be represented by each expression, and then represent each product or quotient as a single rational number.

6. $8 \times (-0.25)$

7. $-6 \div \left(1\frac{1}{3}\right)$

8. $-\frac{1}{2} \times 12$

Lesson 16: Applying the Properties of Operations to Multiply and Divide Rational Numbers

Classwork

Example 1: Using the Commutative and Associative Properties to Efficiently Multiply Rational Numbers

- a. Evaluate the expression below.

$$-6 \times 2 \times (-2) \times (-5) \times (-3)$$

- b. What types of strategies were used to evaluate the expressions?
- c. Can you identify the benefits of choosing one strategy versus another?
- d. What is the sign of the product, and how was the sign determined?

Exercise 1

Find an efficient strategy to evaluate the expression and complete the necessary work.

$$-1 \times (-3) \times 10 \times (-2) \times 2$$

Exercise 2

Find an efficient strategy to evaluate the expression and complete the necessary work.

$$4 \times \frac{1}{3} \times (-8) \times 9 \times \left(-\frac{1}{2}\right)$$

Exercise 3

What terms did you combine first and why?

Exercise 4

Refer to the example and exercises. Do you see an easy way to determine the sign of the product first?

Example 2: Using the Distributive Property to Multiply Rational Numbers

Rewrite the mixed number as a sum; then, multiply using the distributive property.

$$-6 \times \left(5\frac{1}{3}\right)$$

Exercise 5

Multiply the expression using the distributive property.

$$9 \times \left(-3\frac{1}{2}\right)$$

Example 3: Using the Distributive Property to Multiply Rational Numbers

Evaluate using the distributive property.

$$16 \times \left(-\frac{3}{8}\right) + 16 \times \frac{1}{4}$$

Example 4: Using the Multiplicative Inverse to Rewrite Division as Multiplication

Rewrite the expression as only multiplication and evaluate.

$$1 \div \frac{2}{3} \times (-8) \times 3 \div \left(-\frac{1}{2}\right)$$

Exercise 6

$$4.2 \times \left(-\frac{1}{3}\right) \div \frac{1}{6} \times (-10)$$

Name _____

Date _____

Lesson 16: Applying the Properties of Operations to Multiply and Divide Rational Numbers

Exit Ticket

1. Evaluate the expression below using the properties of operations.

$$18 \div \left(-\frac{2}{3}\right) \times 4 \div (-7) \times (-3) \div \left(\frac{1}{4}\right)$$

✦

✦

2.

- a. Given the expression below, what will the sign of the product be? Justify your answer.

$$-4 \times \left(-\frac{8}{9}\right) \times 2.78 \times \left(1\frac{1}{3}\right) \times \left(-\frac{2}{5}\right) \times (-6.2) \times (-0.2873) \times \left(3\frac{1}{11}\right) \times A$$

- b. Give a value for A that would result in a positive value for the expression.

- c. Give a value for A that would result in a negative value for the expression.

Name _____

Date _____

Lesson 16: Applying the Properties of Operations to Multiply and Divide Rational Numbers

Lesson Summary

Multiplying and dividing using the strict order of the operations in an expression is not always efficient. The properties of multiplication allow us to manipulate the expression by rearranging and regrouping factors that are easier to compute (like grouping factors 2 and 5 to get 10).

Where division is involved, we can easily rewrite the division by a number as multiplication by its reciprocal, and then use the properties of multiplication.

If an expression is only a product of factors, then the sign of its value is easily determined by the number of negative factors: the sign is positive if there are an even number of negative factors and negative if there is an odd number of factors.

Problem Set

1. Evaluate the expression $-2.2 \times (-2) \div \left(-\frac{1}{4}\right) \times 5$

a. Using the order of operations only.

b. Using the properties and methods used in Lesson 16.

c. If you were asked to evaluate another expression, which method would you use, (a) or (b), and why?

2. Evaluate the expressions using the distributive property.

a. $\left(2\frac{1}{4}\right) \times (-8)$

b. $\frac{2}{3}(-7) + \frac{2}{3}(-5)$

3. Mia evaluated the expression below but got an incorrect answer. Find Mia's error(s), find the correct value of the expression, and explain how Mia could have avoided her error(s).

$$0.38 \times 3 \div \left(-\frac{1}{20}\right) \times 5 \div (-8)$$

$$0.38 \times 5 \times \left(\frac{1}{20}\right) \times 3 \times (-8)$$

$$0.38 \times \left(\frac{1}{4}\right) \times 3 \times (-8)$$

$$0.38 \times \left(\frac{1}{4}\right) \times (-24)$$

$$0.38 \times (-6)$$

$$-2.28$$

Name: _____

Date: _____ Period: _____

7.2 Mid-Module Review A

Kevin used a number line to add. He started counting at 6, and then he counted until he was on the number -8 on the number line.

1. If Kevin is modeling addition, what number did he add to 6? Use the number line below to model your answer.



2. Write a real-world story problem that would fit this situation.

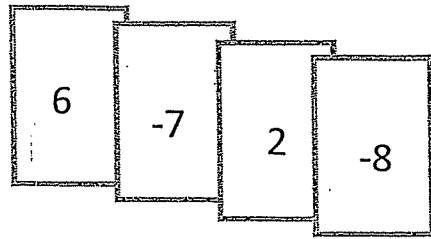
3. Use absolute value to express the distance between 6 and -8.

Every month, Mrs. Jones pays her car loan through automatic payments (withdrawals) from her savings account. She pays the same amount on her car loan each month. At the end of the year, her savings account balance changed by $-\$2,356.80$ from payments made on her car loan.

4. What is the change in Mrs. Jones' savings account balance each month due to her car payment?

5. Describe the total change to Mrs. Jones' savings account balance after making four monthly payments on her car loan. Model your answer using a number sentence.

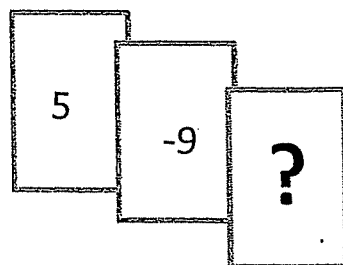
Brooke and Sara are playing the Integer Card Game. The cards in Brooke's hand are shown below.



6. What is the total score of Brooke's hand? Support your answer by showing your work.

7. Brooke picks up two more cards, but they do not affect her overall point total. State the value of each of the two cards, and tell why they do not affect her overall point total.

8. Complete Brooke's new hand to make this total score equal to zero. What must be the value of the ? card? Explain how you arrived at your answer.



9. Michael and Julie were playing the Integer Card Game like the one you played in class. They were practicing adding and subtracting integers. Julie had a score of -15 . Michael took away one of Julie's cards. He showed it to her. It was a -9 . Julie recalculated her score to be -6 , but Michael disagreed. He said that her score should be -24 instead. Read their conversation, and answer the question below.

Michael said:

"No, Julie, removing a negative card means the same thing as subtracting a positive. So, negative 15 minus negative 9 is negative 24."

Julie said:

"It does not! Removing a negative card is the same as adding the same positive card. My score will go up. Negative 15 minus negative 9 is negative 6."

Based on their disagreement, who, if anyone is right? Explain.

The table below shows the temperature changes Monday morning in Chicago, Illinois over a 4-hour period after a cold front came through.

10. If the beginning temperature was -5 at 5:00 AM, what was the temperature at 9:00 AM?

Change in Temperature	
5:00 a.m. – 6:00 a.m.	-3°F
6:00 a.m. – 7:00 a.m.	-2°F
7:00 a.m. – 8:00 a.m.	-6°F
8:00 a.m. – 9:00 a.m.	7°F

11. The same cold front hit Indianapolis, Indiana the next morning. The temperature dropped by 6°F each hour from 5:00 AM to 9:00 AM. What was the beginning temperature at 5:00 AM if the temperature at 9:00 AM was -11°F ?

12. In answering number 11, Mary and Joe used different methods. Mary said her method involved multiplication, while Joe said he did not use multiplication. Both students arrived at the correct answer. How is this possible? Explain.

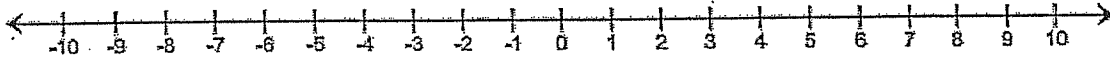
Name: _____

Date: _____ Period: _____

7.2 Mid-Module Review B

Josh used a number line to add. He started counting at 7, and then he counted until he was on the number -5 on the number line.

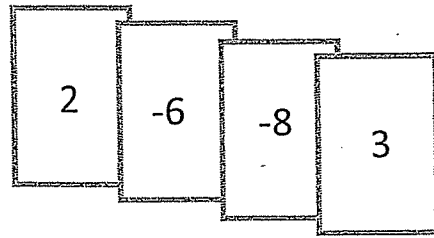
1. If Josh is modeling addition, what number did he add to 7? Use the number line below to model your answer.



2. Write a real-world story problem that would fit this situation.

3. Use absolute value to express the distance between 7 and -5.

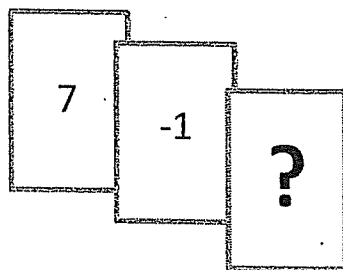
Jamie and Carol are playing the Integer Card Game. The cards in Jamie's hand are shown below.



6. What is the total score of Jamie's hand? Support your answer by showing your work.

7. Jamie picks up two more cards, but they do not affect her overall point total. State the value of each of the two cards, and tell why they do not affect her overall point total.

8. Complete Jamie's new hand to make this total score equal to zero. What must be the value of the ? card? Explain how you arrived at your answer.



9. Steve and Nicole were playing the Integer Card Game like the one you played in class. They were practicing adding and subtracting integers. Nicole had a score of -11 . Steve took away one of Nicole's cards. He showed it to her. It was a -7 . Nicole recalculated her score to be -4 , but Steve disagreed. He said that her score should be -18 instead. Read their conversation, and answer the question below.

Steve said:

"No, Nicole, removing a negative card means the same thing as subtracting a positive. So, negative 11 minus negative 7 is negative 18."

Nicole said:

"It does not! Removing a negative card is the same as adding the same positive card. My score will go up. Negative 11 minus negative 7 is negative 4."

Based on their disagreement, who, if anyone is right? Explain.

The table below shows the temperature changes Monday morning in Milwaukee, Wisconsin over a 4-hour period after a cold front came through.

10. If the beginning temperature was -16 at 5:00 AM, what was the temperature at 9:00 AM?

Change in Temperature	
5:00 a.m. – 6:00 a.m.	-3°F
6:00 a.m. – 7:00 a.m.	-2°F
7:00 a.m. – 8:00 a.m.	-6°F
8:00 a.m. – 9:00 a.m.	7°F

11. The same cold front hit Cleveland, Ohio the next morning. The temperature dropped by 4°F each hour from 5:00 AM to 9:00 AM. What was the beginning temperature at 5:00 AM if the temperature at 9:00 AM was -21°F ?
12. In answering number 11, Lisa and Adam used different methods. Lisa said her method involved multiplication, while Adam said he did not use multiplication. Both students arrived at the correct answer. How is this possible? Explain.

Lesson 17: Comparing Tape Diagram Solutions to Algebraic Solutions

Classwork

Opening Exercise

For his birthday, Zack and three of his friends went to a movie. They each got a ticket for \$8.00 and the same snack from the concession stand. If Zack's mom paid \$48 for the group's tickets and snacks, how much did each snack cost?

The equation $4(s + 8) = 48$ represents the situation when s represents the cost, in dollars, of one snack.

Exploratory Challenge: Expenses on Your Family Vacation

John and Ag are summarizing some of the expenses of their family vacation for themselves and their three children, Louie, Missy, and Bonnie. Write an algebraic equation, create a model to determine how much each item will cost using all of the given information, and answer the questions that follow.

Expenses:

Car and insurance fees: \$400	Airfare and insurance fees: \$875	Motel and tax: \$400
Baseball game and hats: \$103.83	Movies for one day: \$75	Soda and pizza: \$37.95
	Sandals and T-shirts: \$120	

Your Group's Scenario Solution:

Exercise

The cost of a babysitting service on a cruise is \$10 for the first hour and \$12 for each additional hour. If the total cost of babysitting baby Aaron was \$58, how many hours was Aaron at the sitter?

Exploratory Challenge Scenarios

Scenario 1

During one rainy day on the vacation, the entire family decided to go watch a matinee movie in the morning and a drive-in movie in the evening. The price for a matinee movie in the morning is different than the cost of a drive-in movie in the evening. The tickets for the matinee morning movie cost \$6 each. How much did each person spend that day on movie tickets if the ticket cost for each family member was the same? What was the cost for a ticket for the drive-in movie in the evening?

Scenario 2

For dinner one night, the family went to the local pizza parlor. The cost of a soda was \$3. If each member of the family had a soda and one slice of pizza, how much did one slice of pizza cost?

Scenario 3

One night, John, Louie, and Bonnie went to see the local baseball team play a game. They each bought a game ticket and a hat that cost \$10. How much was each ticket to enter the ballpark?

Scenario 4

While John, Louie, and Bonnie went to see the baseball game, Ag and Missy went shopping. They bought a T-shirt for each member of the family and bought two pairs of sandals that cost \$10 a pair. How much was each T-shirt?

Scenario 5

The family flew in an airplane to their vacation destination. Each person had to have his own ticket for the plane and also pay \$25 in insurance fees per person. What was the cost of one ticket?

Scenario 6

While on vacation, the family rented a car to get them to all the places they wanted to see for five days. The car costs a certain amount each day, plus a one-time insurance fee of \$50. How much was the daily cost of the car (not including the insurance fees)?

Scenario 7

The family decided to stay in a motel for four nights. The motel charges a nightly fee plus \$60 in state taxes. What is the nightly charge with no taxes included?

Name _____

Date _____

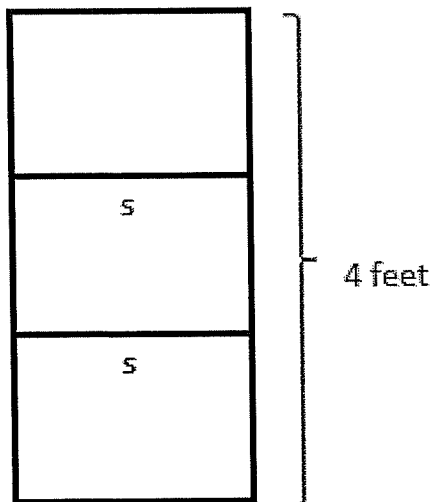
Lesson 17: Comparing Tape Diagram Solutions to Algebraic Solutions

Solutions

Exit Ticket

1. Eric's father works two part-time jobs, one in the morning and one in the afternoon, and works a total of 40 hours each 5-day workweek. If his schedule is the same each day, and he works 3 hours each morning, how many hours does Eric's father work each afternoon?

2. Henry is using a total of 16 ft. of lumber to make a bookcase. The left and right sides of the bookcase are each 4 ft. high. The top, bottom, and two shelves are all the same length, labeled S . How long is each shelf?



Name _____

Date _____

Lesson 17: Comparing Tape Diagram Solutions to Algebraic Solutions

Lesson Summary

Tape diagrams can be used to model and identify the sequence of operations to find a solution algebraically.

The goal in solving equations algebraically is to isolate the variable.

The process of doing this requires *undoing* addition or subtraction to obtain a 0 and *undoing* multiplication or division to obtain a 1. The additive inverse and multiplicative inverse properties are applied to get the 0 (the additive identity) and 1 (the multiplicative identity).

The addition and multiplication properties of equality are applied because in an equation, $A = B$, when a number is added or multiplied to both sides, the resulting sum or product remains equal.

Problem Set

1. A taxi cab in Myrtle Beach charges \$2 per mile and \$1 for every person. If a taxi cab ride for two people costs \$12, how far did the taxi cab travel?

2. Heather works as a waitress at her family's restaurant. She works 2 hours every morning during the breakfast shift and returns to work each evening for the dinner shift. In the last four days, she worked 28 hours. If Heather works the same number of hours every evening, how many hours did she work during each dinner shift?

3. Jillian exercises 5 times a week. She runs 3 miles each morning and bikes in the evening. If she exercises a total of 30 miles for the week, how many miles does she bike each evening?

4. Marc eats an egg sandwich for breakfast and a big burger for lunch every day. The egg sandwich has 250 calories. If Marc has 5,250 calories for breakfast and lunch for the week in total, how many calories are in one big burger?

5. Jackie won tickets playing the bowling game at the local arcade. The first time, she won 60 tickets. The second time, she won a bonus, which was 4 times the number of tickets of the original second prize. Altogether she won 200 tickets. How many tickets was the original second prize?

Lesson 18: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Classwork

Exercise 1

John's father asked him to compare several different cell phone plans and identify which plan will be the least expensive for the family. Each phone company charges a monthly fee, but this fee does not cover any services: phone lines, texting, or internet access. Use the information contained in the table below to answer the following questions.

Cell Phone Plans

Name of Plan	Monthly Fee (Includes 1,500 shared minutes)	Price per Phone Line x	Price per line for Unlimited Texting y	Price per line for Internet Access z
Company A	\$70	\$20	\$15	\$15
Company B	\$90	\$15	\$10	\$20
Company C	\$200	\$10	included in monthly fee	included in monthly fee

All members of the family may not want identical plans; therefore, we will let x represent the number of phone lines, y represent the number of phone lines with unlimited texting, and z represent the number of phone lines with internet access.

Expression

Company A _____

Company B _____

Company C _____

Using the expressions above, find the cost to the family of each company's phone plan if:

- a. Four people want a phone line, four people want unlimited texting, and the family needs two internet lines.

Company A	Company B	Company C

Which cell phone company should John's family use? Why?

- b. Four people want a phone line, four people want unlimited texting, and all four people want internet lines.

Company A	Company B	Company C

Which cell phone company should John's family use? Why?

- c. Two people want a phone line, two people want unlimited texting, and the family needs two internet lines.

Company A	Company B	Company C

Which cell phone company should John's family use? Why?

Exercise 2

Three friends went to the movies. Each purchased a medium-sized popcorn for p dollars and a small soft drink for s dollars.

- Write the expression that represents the total amount of money (in dollars) the three friends spent at the concession stand.

- If the concession stand charges \$6.50 for a medium-sized popcorn and \$4.00 for a small soft drink, how much did the three friends spend on their refreshments altogether?

Exercise 3

Complete the table below by writing equivalent expressions to the given expression and evaluating each expression with the given values.

Equivalent Expressions			
EXAMPLE: Evaluate $x = 2,$ $y = -1$	$4(x + 2y)$ $4(2 + 2(-1))$ $4(0)$ 0	$4x + 8y$ $4(2) + 8(-1)$ $8 + (-8)$ 0	$4x + 4y + 4y$ $4(2) + 4(-1) + 4(-1)$ $8 + (-4) + (-4)$ 0
1. Evaluate $y = 1$	$5(3 - 4y)$		
2. Evaluate $x = 5,$ $y = -2$	$-3x + 12y$		

3. Evaluate $x = -\frac{1}{2}$, $y = 1$			$-2x + 10x - 6y$
--	--	--	------------------

Name _____

Date _____

Lesson 18: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Exit Ticket

Bradley and Louie are roommates at college. At the beginning of the semester, they each paid a security deposit of A dollars. When they move out, their landlord will deduct from this deposit any expenses (B) for excessive wear and tear and refund the remaining amount. Bradley and Louie will share the expenses equally.

- Write an expression that describes the amount each roommate will receive from the landlord when the lease expires.
- Evaluate the expression using the following information: Each roommate paid a \$125 deposit, and the landlord deducted \$50 total for damages.

Name _____

Date _____

Lesson 18: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Lesson Summary

- An expression is a number or a letter, which can be raised to a whole number exponent. An expression can be a product whose factors are any one of the entities described above. An expression can also be the sum or difference of the products described above.
- To evaluate an expression, replace each variable with its corresponding numerical value. Using order of operations, the expression can be written as a single numerical value.
- When numbers are substituted into all the letters in an expression and the results are the same, then the expressions are equivalent.

Problem Set

1. Sally is paid a fixed amount of money to walk her neighbor's dog every day after school. When she is paid each month, she puts aside \$20 to spend and saves the remaining amount. Write an expression that represents the amount Sally will save in 6 months if she earns m dollars each month. If Sally is paid \$65 each month, how much will she save in 6 months?

4. Profit is defined as earnings less expenses (earnings $-$ expenses). At the local hot-air balloon festival, the Ma & Pops Ice Cream Truck sells ice cream pops, which cost them \$0.75 each, but are sold for \$2 each. They also paid \$50 to the festival's organizers for a vendor permit. The table below shows the earnings, expenses, and profit earned when 50, 75, and 100 ice cream pops were sold at the festival.

Number of Pops Sold	Earnings	Expenses	Profit
50	$50(2) = 100$	$50(0.75) + 50$ $37.5 + 50 = 87.5$	$100 - 87.5 = 12.50$
75	$75(2) = 150$	$75(0.75) + 50$ $56.25 + 50 = 106.25$	$150 - 106.25 = 43.75$
100	$100(2) = 200$	$100(0.75) + 50$ $75 + 50 = 125$	$200 - 125 = 75$

- a. Write an expression that represents the profit (in dollars) Ma & Pops earned by selling ice cream pops at the festival.
- b. Write an equivalent expression.
- c. How much of a profit did Ma & Pops Ice Cream Truck make if it sold 20 ice cream pops? What does this mean? Explain why this might be the case.
- d. How much of a profit did Ma & Pops Ice Cream Truck make if it sold 75 ice cream pops? What does this mean? Explain why this might be the case.

Lesson 19: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Classwork

Example 1: Tic-Tac-Toe Review

Fill in the 9 spaces with one expression from the list below. Use one expression per space. You will use 9 of the expressions:

$$12 - 4x$$

$$8x + 4 - 12x$$

$$8\left(\frac{1}{2}x - 2\right)$$

$$12 - 6x + 2x$$

$$-4x + 4$$

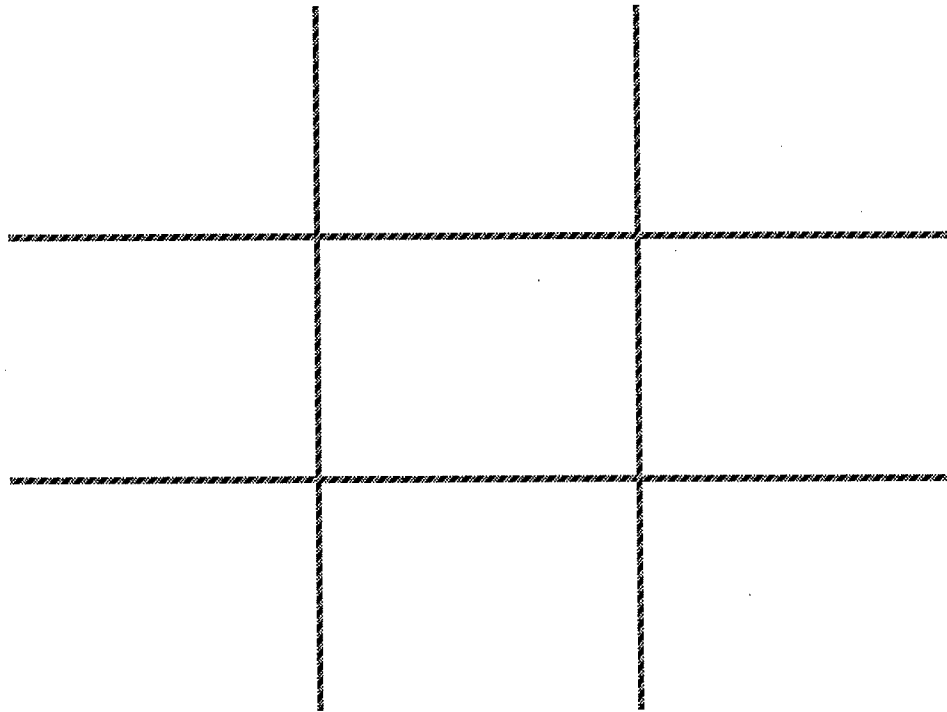
$$x - 2 + 2x - 4$$

$$4x - 12$$

$$4(x - 4)$$

$$3(x - 2)$$

$$0.1(40x) - \frac{1}{2}(24)$$



Example 2

Original Price (100%)	Discount Amount (20% Off)	New Price (Pay 80%)	Expression
100			
50			
28			
14.50			
x			

Example 4

Original Price (100%)	Discount (20% off)	Amount Pay (pay 80%)	Expression	New Price	Sales Tax (8%)	Overall Cost	Expression
100	20	80	$100 - 100(0.20) = 100(0.80)$				
50	10	40	$50 - 50(0.20) = 50(0.80)$				
28	5.60	22.40	$28 - 28(0.20) = 28(0.80)$				
14.50	2.90	11.60	$14.50 - 14.50(0.20)$ or $14.50(0.80)$				
x	$0.20x$	$x - 0.20x$	$x - 0.20x$ or $0.80x$				

Name _____

Date _____

Lesson 19: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Exit Ticket

1. Write three equivalent expressions that can be used to find the final price of an item costing g dollars that is on sale for 15% off and charged 7% sales tax.
2. Using all of the expressions, determine the final price for an item that costs \$75. If necessary, round to the nearest penny.
3. If each expression yields the same final sale price, is there anything to be gained by using one over the other?
4. Describe the benefits, special characteristics, and properties of each expression.

Name _____

Date _____

Lesson 19: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Lesson Summary

- Two expressions are equivalent if they yield the same number for every substitution of numbers for the letters in each expression.
- The expression that allows us to find the cost of an item after the discount has been taken and the sales tax has been added is written by representing the discount price added to the discount price multiplied by the sales tax rate.

Problem Set

Solve the following problems. If necessary, round to the nearest penny.

1. A family of 12 went to the local Italian restaurant for dinner. Every family member ordered a drink and meal, 3 ordered an appetizer, and 6 people ordered cake for dessert.
 - a. Write an expression that can be used to figure out the cost of the bill. Include the definitions for the variables the server used.
 - b. The waitress wrote on her ordering pad the following expression: $3(4d + 4m + a + 2c)$. Was she correct? Explain why or why not.
 - c. What is the cost of the bill if a drink costs \$3, a meal costs \$20, an appetizer costs \$5.50, and a slice of cake costs \$3.75?
 - d. Suppose the family had a 10% discount coupon for the entire check and then left an 18% tip. What is the total?

2. Sally designs web pages for customers. She charges \$135.50 per web page; however, she must pay a monthly rental fee of \$650 for her office. Write an expression to determine her take-home pay after expenses. If Sally designed 5 web pages last month, what was her take-home pay after expenses?

3. While shopping, Megan and her friend Rylie find a pair of boots on sale for 25% off the original price. Megan calculates the final cost of the boots by first deducting the 25% and then adding the 6% sales tax. Rylie thinks Megan will pay less if she pays the 6% sales tax first and then takes the 25% discount.

a. Write an expression to represent each girl's scenario if the original price of the boots was x dollars.

b. Evaluate each expression if the boots originally cost \$200.

c. Who was right? Explain how you know.

d. Explain how both girls' expressions are equivalent.

Lesson 20: Investments—Performing Operations with Rational Numbers

Classwork

Mathematical Modeling Exercise: College Investments

Justin and Adrienne deposited \$20,000 into an investment account for 5 years. They hoped the money invested and the money made on their investment would amount to at least \$30,000 to help pay for their daughter's college tuition and expenses. The account they chose has several benefits and fees associated with it. Every 6 months, a summary statement is sent to Justin and Adrienne. The statement includes the amount of money either gained or lost. Below are semiannual (twice a year) statements for a period of 5 years. In addition to the statements, the following information is needed to complete the task:

- For every statement, there is an administrative fee of \$15 to cover costs such as secretarial work, office supplies, and postage.
- If there is a withdrawal made, a broker's fee is deducted from the account. The amount of the broker's fee is 2% of the transaction amount.

TASK: Using the above information, semiannual statements, register, and beginning balance, do the following:

1. Record the beginning balance and all transactions from the account statements into the register.
2. Determine the annual gain or loss as well as the overall 5-year gain or loss.
3. Determine if there is enough money in the account after 5 years to cover \$30,000 of college expenses for Justin and Adrienne's daughter. Write a summary to defend your answer. Be sure to indicate how much money is in excess, or the shortage that exists.
4. Answer the related questions that follow.

College Investment Fund Semi-Annual Statement

January 1, 2008 – June 30, 2008

Investment Gain/(Loss): 700.00

College Investment Fund Semi-Annual Statement

July 1, 2008 – December 31, 2008

Investment Gain/(Loss): 754.38

College Investment Fund Semi-Annual Statement

January 1, 2009 – June 30, 2009

Investment Gain/(Loss): (49.88)

College Investment Fund Semi-Annual Statement

July 1, 2009 – December 31, 2009

Withdrawal: 500.00
Investment Gain/(Loss): (17.41)

College Investment Fund Semi-Annual Statement

January 1, 2010 – June 30, 2010

Investment Gain/(Loss): 676.93

College Investment Fund Semi-Annual Statement

July 1, 2010 – December 31, 2010

Investment Gain/(Loss): 759.45

College Investment Fund Semi-Annual Statement

January 1, 2011 – June 30, 2011

Deposit: 1,500.00
Investment Gain/(Loss): 880.09

College Investment Fund Semi-Annual Statement

July 1, 2011 – December 31, 2011

Investment Gain/(Loss): 922.99

College Investment Fund Semi-Annual Statement

January 1, 2012 – June 30, 2012

Deposit: 800.00
Investment Gain/(Loss): 942.33

College Investment Fund Semi-Annual Statement

July 1, 2012 – December 31, 2012

Investment Gain/(Loss): 909.71

5. Register

DATE	DESCRIPTION OF TRANSACTION	WITHDRAWAL	DEPOSIT	BALANCE	EXPRESSION
	Beginning Balance	---	---	\$20,000.00	\$20,000.00
Jan. – June: 2008					
July – Dec.: 2008					
Jan. – June: 2009					
July – Dec.: 2009					
Jan. – June: 2010					
July – Dec.: 2010					
Jan. – June: 2011					
July – Dec.: 2011					
Jan. – June: 2012					
July – Dec.: 2012					

6. Annual Gain/Loss Summary

Year	Total Gain/(Loss)	Numerical Expression
2008		
2009		
2010		
2011		
2012		
5-Year Gain/Loss		

7. Summary

Exercise

Below is a transaction log of a business entertainment account. The transactions are completed and the ending balance in the account is \$525.55. Determine the beginning balance.

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	
12/1/10	Bargain Electronic (i-Pod)	199.99		
12/5/10	Lenny's Drive-Up (Gift Certificate)	75.00		
12/7/10	Check from Customer: Reynolds		200.00	
12/15/10	Pasta House (Dinner)	285.00		
12/20/10	Refund from Clear's Play House		150.00	
12/22/10	Gaffney's Tree Nursery	65.48		525.55

Name _____

Date _____

Lesson 20: Investments—Performing Operations with Rational Numbers

Lesson Summary

- Calculations with rational numbers are used when recording investment transactions.
- Deposits are added to an account balance; money is deposited into the account.
- Gains are added to an account balance; they are positive returns on the investment.
- Withdrawals are subtracted from an account balance; money is taken out of the account.
- Losses are subtracted from an account balance; they are negative returns on the investment.
- Fees are subtracted from an account balance; the bank or financial company is charging you for a service.

Problem Set

1. You are planning a fundraiser for your student council. The fundraiser is a Glow in the Dark Dance. Solve each entry below, and complete the transaction log to determine the ending balance in the student account.
 - a. The cost of admission to the dance is \$7 per person, and all tickets were sold on November 1. Write an expression to represent the total amount of money collected for admission. Evaluate the expression if 250 people attended the dance.
 - b. The following expenses were necessary for the dance, and checks were written to each company.
 - DJ for the dance—*Music Madness DJ* costs \$200 and paid for on November 3.
 - Glow sticks from *Glow World, Inc.* for the first 100 entrants. Cost of glow sticks was \$0.75 each plus 8% sales tax and bought on November 4.

Complete the transaction log below based on this information

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	1,243.56

- c. Write a numerical expression to determine the cost of the glow sticks.

Analyze the results.

- d. Write an algebraic expression to represent the profit earned from the fundraiser. (Profit is the amount of money collected in admissions minus all expenses.)
- e. Evaluate the expression to determine the profit if 250 people attended the dance. Use the variable p to represent the number of people attending the dance (from part (a)).
- f. Using the transaction log above, what was the amount of the profit earned?

2. The register below shows a series of transactions made to an investment account. Vinnie and Anthony both completed the register in hopes of finding the beginning balance. As you can see, they do not get the same answer. Who was correct? What mistake did the other person make? What was the monthly gain or loss?

Original Register

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	
3/1/11	Broker's Fee	250.00		
3/10/11	Loan Withdrawal	895.22		
3/15/11	Refund – Misc. Fee		50.00	
3/31/11	Investment Results		2,012.22	18,917.00

Vinnie's Work

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	18,000.00
3/1/11	Broker's Fee	250.00		17,750.00
3/10/11	Loan Withdrawal	895.22		16,854.78
3/15/11	Refund – Misc. Fee		50.00	16,904.78
3/31/11	Investment Results		2,012.22	18,917.00

Anthony's Work

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	19,834.00
3/1/11	Broker's Fee	250.00		20,084.00
3/10/11	Loan Withdrawal	895.22		20,979.22
3/15/11	Refund – Misc. Fee		50.00	20,929.22
3/31/11	Investment Results		2,012.22	18,917.00

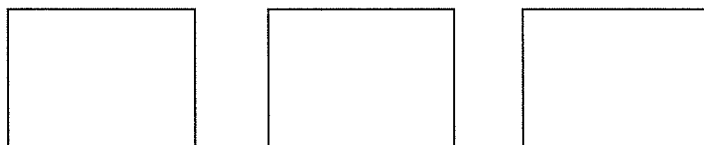
Lesson 21: If–Then Moves with Integer Number Cards

Classwork

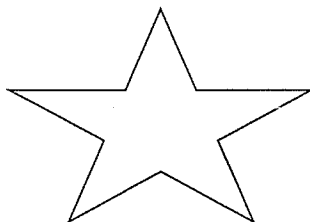
Exploratory Challenge: Integer Game Revisited

Let's investigate what happens if a card is added or removed from a hand of integers.

My cards:

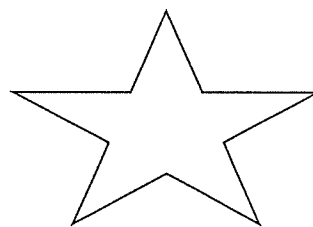
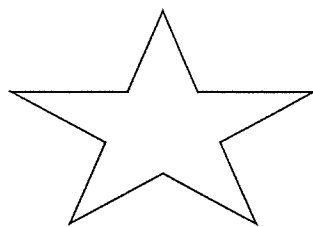


My score:



Event 1

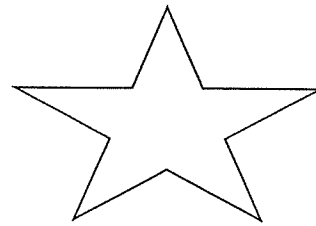
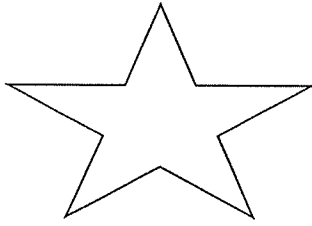
My new score:



Conclusion:

Event 2

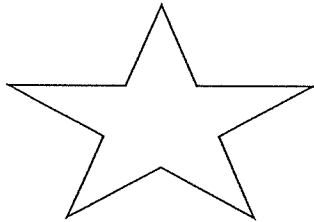
My new score:



Conclusion:

Event 3

My new score:



Expression:

Conclusion:

Event 4

Expression:

Conclusion:

Exercises

1. The table below shows two hands from the Integer Game and a series of changes that occurred to each hand. Part of the table is completed for you. Complete the remaining part of the table; then summarize the results.

	Hand 1	Result	Hand 2	Result
Original	$1 + (-4) + 2$		$0 + 5 + (-6)$	
Add 4	$1 + (-4) + 2 + 4$			
Subtract 1	$1 + (-4) + 2 + 4 - 1$			
Multiply by 3				
Divide by 2				

2. Complete the table below using the multiplication property of equality.

	Original expression and result	Equivalent expression and result
	$3 + (-5) =$	
Multiply both expressions by -3		
Write a conclusion using if-then		

Name _____

Date _____

Lesson 21: If–Then Moves with Integer Number Cards

Lesson Summary

- If a number sentence is true, and the same number is added to both sides of the equation, then the resulting number sentence is true. (*addition property of equality*)
- If a number sentence is true, and the same number is subtracted from both sides of the equation, then the resulting number sentence is true. (*subtraction property of equality*)
- If a number sentence is true, and both sides of the equation are multiplied by the same number, then the resulting number sentence is true. (*multiplication property of equality*)
- If a number sentence is true, and both sides of the equation are divided by the same nonzero number, then the resulting number sentence is true. (*division property of equality*)

Problem Set

1. Evaluate the following numerical expressions.

a. $2 + (-3) + 7$

b. $-4 - 1$

c. $-\frac{5}{2} \times 2$

d. $-10 \div 2 + 3$

e. $\left(\frac{1}{2}\right)(8) + 2$

f. $3 + (-4) - 1$

2. Which expressions from Exercise 1 are equal?

3. If two of the equivalent expressions from Exercise 1 are divided by 3, write an if-then statement using the properties of equality.

4. Write an if-then statement if -3 is multiplied to the following equation: $-1 - 3 = -4$.

5. Simplify the expression.

$$5 + 6 - 5 + 4 + 7 - 3 + 6 - 3$$

Using the expression, write an equation.

Rewrite the equation if 5 is added to both expressions.

Write an if-then statement using the properties of equality.

Lesson 22: Solving Equations Using Algebra

Classwork

In this lesson, you will transition from solving equations using tape diagrams to solving equations algebraically by *making zero* (using the additive inverse) and *making one* (using the multiplicative inverse). Justify your work by identifying which algebraic property you used for each step in solving the problems. Explain your work by writing out how you solved the equations step by step and relate each step to those used with a tape diagram.

Example 1: Yoshiro's New Puppy

Yoshiro has a new puppy. She decides to create an enclosure for her puppy in her backyard. The enclosure is in the shape of a hexagon (six-sided polygon) with one pair of opposite sides running the same distance along the length of two parallel flower beds. There are two boundaries at one end of the flower beds that are 10 ft. and 12 ft., respectively, and at the other end, the two boundaries are 15 ft. and 20 ft., respectively. If the perimeter of the enclosure is 137 ft., what is the length of each side that runs along the flower bed?

Example 2: Swim Practice

Jenny is on the local swim team for the summer and has swim practice four days per week. The schedule is the same each day. The team swims in the morning and then again for 2 hours in the evening. If she swims 12 hours per week, how long does she swim each morning?

Exercises

Solve each equation algebraically using if–then statements to justify each step.

1. $5x + 4 = 19$

2. $15x + 14 = 19$

3. Claire’s mom found a very good price on a large computer monitor. She paid \$325 for a monitor that was only \$65 more than half the original price. What was the original price?

4. $2(x + 4) = 18$

5. Ben's family left for vacation after his dad came home from work on Friday. The entire trip was 600 mi. Dad was very tired after working a long day and decided to stop and spend the night in a hotel after 4 hours of driving. The next morning, Dad drove the remainder of the trip. If the average speed of the car was 60 miles per hour, what was the remaining time left to drive on the second part of the trip? Remember: Distance = rate multiplied by time.

Name _____

Date _____

Lesson 22: Solving Equations Using Algebra

Exit Ticket

Susan and Bonnie are shopping for school clothes. Susan has \$50 and a coupon for a \$10 discount at a clothing store where each shirt costs \$12.

Susan thinks that she can buy three shirts, but Bonnie says that Susan can buy five shirts. The equations they used to model the problem are listed below. Solve each equation algebraically, justify your steps, and determine who is correct and why.

Susan's Equation

$$12n + 10 = 50$$

Bonnie's Equation

$$12n - 10 = 50$$

Name _____

Date _____

Lesson 22: Solving Equations Using Algebra

Lesson Summary

We work backward to solve an algebraic equation. For example, to find the value of the variable in the equation $6x - 8 = 40$:

1. Use the addition property of equality to add the opposite of -8 to each side of the equation to arrive at $6x - 8 + 8 = 40 + 8$.
2. Use the additive inverse property to show that $-8 + 8 = 0$; thus, $6x + 0 = 48$.
3. Use the additive identity property to arrive at $6x = 48$.
4. Then use the multiplication property of equality to multiply both sides of the equation by $\frac{1}{6}$ to get:

$$\left(\frac{1}{6}\right) 6x = \left(\frac{1}{6}\right) 48.$$

5. Then use the multiplicative inverse property to show that $\frac{1}{6}(6) = 1$; thus, $1x = 8$.
6. Use the multiplicative identity property to arrive at $x = 8$.

Problem Set

For each problem below, explain the steps in finding the value of the variable. Then find the value of the variable, showing each step. Write if-then statements to justify each step in solving the equation.

1. $7(m + 5) = 21$

2. $-2v + 9 = 25$

3. $\frac{1}{3}y - 18 = 2$

4. $6 - 8p = 38$

5. $15 = 5k - 13$

Suppose you want to buy your favorite ice cream bar while at the amusement park and it costs \$2.89. If you purchase the ice cream bar and 3 bottles of water, pay with a \$10 bill, and receive no change, then how much did each bottle of water cost?

- d. Write an equation to model this situation.
- e. Solve the equation to determine the cost of one water bottle. Then write the reason that justifies each step using if-then statements.
- f. Model the problem using a tape diagram to check your work.

- d. Charlotte's goal is to save \$100 for her beach trip at the end of the summer. Use the amount of weekly allowance you found in part (c) to write an equation to determine the number of weeks that Charlotte must work to meet her goal. Let w represent the number of weeks.
- e. In looking at your answer to part (d) and based on the story above, do you think it will take Charlotte that many weeks to meet her goal? Why or why not?

3. Travel Baseball Team

Allen is very excited about joining a travel baseball team for the fall season. He wants to determine how much money he should save to pay for the expenses related to this new team. Players are required to pay for uniforms, travel expenses, and meals.

- a. If Allen buys 4 uniform shirts at one time, he gets a \$10.00 discount so that the total cost of 4 shirts would be \$44. Write an algebraic equation that represents the regular price of one shirt. Solve the equation. Write the reason that justifies each step using if-then statements.

- b. What is the cost of one shirt without the discount?
- c. What is the cost of one shirt with the discount?
- d. How much more do you pay per shirt if you buy them one at a time (rather than in bulk)?

Allen's team was also required to buy two pairs of uniform pants and two baseball caps, which total \$68. A pair of pants costs \$12 more than a baseball cap.

- e. Write an equation that models this situation. Let c represent the cost of a baseball cap.

- f. Solve the equation algebraically to find the cost of a baseball cap. Write the reason that justifies each step using if-then statements.
- g. Model the problem using a tape diagram in order to check your work from part (f).
- h. What is the cost of one cap?
- i. What is the cost of one pair of pants?

Name _____

Date _____

Lesson 23: Solving Equations Using Algebra

Exit Ticket

Andrew's math teacher entered the seventh-grade students in a math competition. There was an enrollment fee of \$30 and also an \$11 charge for each packet of 10 tests. The total cost was \$151. How many tests were purchased?

Set up an equation to model this situation, solve it using if-then statements, and justify the reasons for each step in your solution.

Name _____

Date _____

Lesson 23: Solving Equations Using Algebra

Lesson Summary

Equations are useful to model and solve real-world problems. The steps taken to solve an algebraic equation are the same steps used in an arithmetic solution.

Problem Set

For Exercises 1–4, solve each equation algebraically using if-then statements to justify your steps.

1. $\frac{2}{3}x - 4 = 20$

2. $4 = \frac{-1+x}{2}$

3. $12(x + 9) = -108$

4. $5x + 14 = -7$

For Exercises 5–7, write an equation to represent each word problem. Solve the equation showing the steps and then state the value of the variable in the context of the situation.

5. A plumber has a very long piece of pipe that is used to run city water parallel to a major roadway. The pipe is cut into two sections. One section of pipe is 12 ft. shorter than the other. If $\frac{3}{4}$ of the length of the shorter pipe is 120 ft, how long is the longer piece of the pipe?

6. Bob's monthly phone bill is made up of a \$10 fee plus \$0.05 per minute. Bob's phone bill for July was \$22. Write an equation to model the situation using m to represent the number of minutes. Solve the equation to determine the number of phone minutes Bob used in July.

7. Kym switched cell phone plans. She signed up for a new plan that will save her \$3.50 per month compared to her old cell phone plan. The cost of the new phone plan for an entire year is \$294. How much did Kym pay per month under her old phone plan?

2. Julie's mother taught her how to make handmade bracelets to sell at a craft fair. Julie rented a table at the fair for \$20 and set up her work station. Each bracelet that she makes costs approximately \$1.25 for materials. She sells each bracelet for \$5.00.

a. If x represents the number of bracelets sold at the craft fair, which of the following expressions would represent Julie's profit? (Circle all choices that apply.)

i. $-20 + 5x - 1.25x$

ii. $5x - 20 - 1.25x$

iii. $5x - 20$

iv. $2.75x - 20$

v. $3.75x - 20$

b. Julie does not want to lose money on her business. Her mother told her she needs to sell enough bracelets to at least cover her expenses (costs for materials and table rental). Julie figures that if she sells 5 bracelets, she covers her expenses and does not lose any money. Do you agree? Explain and show work to support your answer.

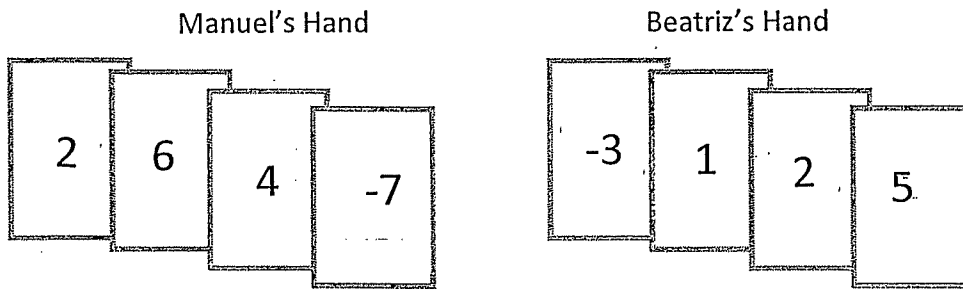
c. Julie feels that if she earns a profit of \$40.00 at this craft fair, her business will be successful enough for her to branch out to other craft fairs. How many bracelets does she have to sell to earn a \$40.00 profit? Write and solve an equation; then explain how the steps and operations used in your algebraic solution compare to an arithmetic solution.

3. John received a letter from his bank saying that his checking account balance fell below zero. His account transaction log is shown below.

Check No.	Date	Description of Transaction	Payment	Deposit	Balance	
	8/12	Beginning Balance			\$438.20	
1124	8/13	Best Buy (Headphones)	\$49.50		-49.50	
					\$388.70	Line 1
1125	8/18	Sports Authority (Shoes)	\$67.00		-67.00	
					\$321.70	Line 2
Debit	8/21	Jewel	\$158.75		-158.75	
					\$162.95	Line 3
1126	8/22	B&B Bike Shop	\$174.45		-174.45	
					\$11.50	Line 4
	8/22	Deposit (Birthday Money)		\$30.00	+30.00	
					\$41.50	Line 5
Debit	8/27	Sal's Sandwich Shop	\$8.35		-8.35	
					\$33.15	Line 6

- a. On which line did John make a mathematical error? Explain John's mistake.
- b. The bank charged John a \$20 fee because his balance dropped below 0. He knows that he currently has an outstanding charge for \$12.60 that he has not recorded yet. How much money will John have to deposit into his account so that the outstanding charge does not create another bank fee? Explain.

4. Manuel and Beatriz are playing the Integer Card Game. The cards in their hands are shown below:



- a. What are the scores in each of their hands?
- b. Kelly says that if Manuel and Beatriz both take away their 2's, Manuel's score will be higher than Beatriz's. Jordan argues and says that Manuel and Beatriz's scores will be equal. Are either of them right? Explain.
- c. Manuel picks up another set of cards that is exactly like each card in his hand. Which of the following would make Beatriz's score equal to Manuel's? Place an X by all that apply.

_____ Double every card in her hand

_____ Take away her 1 and 2

_____ Pick up a 5

_____ Take away her -3 and 5

_____ Pick up a 9 and -4

_____ Pick up one of each of

Manuel's cards

Explain why your selections will make Manuel and Beatriz's scores equal.

2. Betty's mother taught her how to make handmade candles to sell at a craft fair. Betty rented a table at the fair for \$40 and set up her work station. Each candle that she makes costs approximately \$4.75 for materials. She sells each candle for \$10.00.
- a. If x represents the number of candles sold at the craft fair, which of the following expressions would represent Betty's profit? (Circle all choices that apply.)
- i. $10x - 40$
 - ii. $10x - 40 - 4.75x$
 - iii. $-40 + 10x - 4.75x$
 - iv. $5.25x - 40$
 - v. $6.25x - 40$
- b. Betty does not want to lose money on her business. Her mother told her she needs to sell enough candles to at least cover her expenses (costs for materials and table rental). Betty figures that if she sells 7 candles, she covers her expenses and does not lose any money. Do you agree? Explain and show work to support your answer.
- c. Betty feels that if she earns a profit of \$65.00 at this craft fair, her business will be successful enough for her to branch out to other craft fairs. How many candles does she have to sell to earn a \$65.00 profit? Write and solve an equation; then explain how the steps and operations used in your algebraic solution compare to an arithmetic solution.

3. John received a letter from his bank saying that his checking account balance fell below zero. His account transaction log is shown below.

Check No.	Date	Description of Transaction	Payment	Deposit	Balance	
	8/12	Beginning Balance			\$324.30	
1124	8/13	Radio Shack (Cell Phone)	\$279.00		-279.00	
					\$45.30	Line 1
1125	8/18	Kohls (Clothes)	\$67.50		-67.50	
					\$22.20	Line 2
Debit	8/21	Walmart	\$11.85		-11.85	
					\$10.35	Line 3
1126	8/22	Joe's Sub Place	\$7.15		-7.15	
					\$3.20	Line 4
	8/22	Deposit (Mowing Lawns)		\$84.00	+84.00	
					\$87.20	Line 5
Debit	8/27	Lickety Split Ice Cream Shop	\$2.35		-2.35	
					\$84.85	Line 6

- a. On which line did John make a mathematical error? Explain John's mistake.
- b. The bank charged John a \$30 fee because his balance dropped below 0. He knows that he currently has an outstanding charge for \$29.50 that he has not recorded yet. How much money will John have to deposit into his account so that the outstanding charge does not create another bank fee? Explain.

